Gernot Hoffmann

Video tracking, using a single calibrated camera

Tracking objects

Table of Contents

Go to Chapter by clicking on the line
Go to Table of Contents by clicking on the top or bottom stripe of the page

1. Task description

A rectangle with known edge lengths 2a, 2b and corners P_1 to P_4 moves in front of a calibrated camera. The corners are identified in the image, delivering coordinates f_1, g_1 to f_4, g_4. The task is to calculate the position and the three Euler angles of the rectangle, primarily in camera coordinates and, if necessary, in a ground based system. The method is valid for other known objects, if three or more points are identified. The rectangle can be interpreted as a bounding quadrilateral of a face, moving in front of a monitor.

2. Coordinate systems

All coordinate systems are right-handed. The axes are parallel as long as the Euler angles are zero. All axes are using the same scale and the same units, for instance meters, centimeters or millimeters.
The orientation of the axes is different in previous publications by the author, e.g. [4]-[7]. The optical axis of the camera is now the third axis, which is more convenient for applications of homogeneous coordinates (chapter 6).

2.1 x,y,z is the global or base system, for instance fixed to a monitor. When it is fixed in the center of the monitor, x points to the left, y upward and z perpendicular out of the monitor in forward direction, viewing from behind the monitor.

2.2 u,v,w is the camera system. It is centered at $\mathbf{c}=(c_x,c_y,c_z)^T$. The superscript T means *transposed*. Vectors are primarily defined as columns. u points to the left, v upward and w on the optical axes in view direction.

The camera can be rotated by three angles relativ to the base system x,y,z. All angles count positive about the respective axis in the sense of a right screw. See illustration Fig.2 next page.

1. Azimuth β about the vertical axis
2. Elevation α about the new lateral axis
3. Roll angle γ about the optical axis

If the camera is fixed at the upper edge of the monitor in center position, then we have $\mathbf{c}=(0,c_y,0)^T$ and the angles can be considered as small errors of alignment. But one may have as well the situation that the camera is on purpose somewhat tilted by an elevation angle in order to improve the field of view.

2.3 f,g,h is the coordinate system for the image plane, as well represented by three axes.
The intrinsic image plane f,g in our pin hole camera model is placed perpendicularly to the optical axis, shifted by distance d from the center of projection, but in reality backward. d is the camera constant or *Kammerkonstante* – not a *calibrated focal length*, because a pin hole camera does not have a focal length. The third coordinate h completes the 3D-system.

2.4 k,l,m is the object or body system.
It is centered at $\mathbf{b}=(b_u,b_v,b_w)^T$ in the camera system and it can be rotated relative to the camera system by three angles:

1. Yaw angle ψ about the vertical axis
2. Pitch angle θ about the new lateral axis
3. Roll angle ϕ about the new longitudinal axis

The observed object is defined in this system. General object points P_j may be used, but later a rectangle will be defined by $P_1=(a,b,0)$, $P_2=(-a,b,0)$, $P_3=(-a,-b,0)$, $P_4=(a,-b,0)$. The adjoint vectors are $\mathbf{k}_1=(a,b,0)^T$ etc.

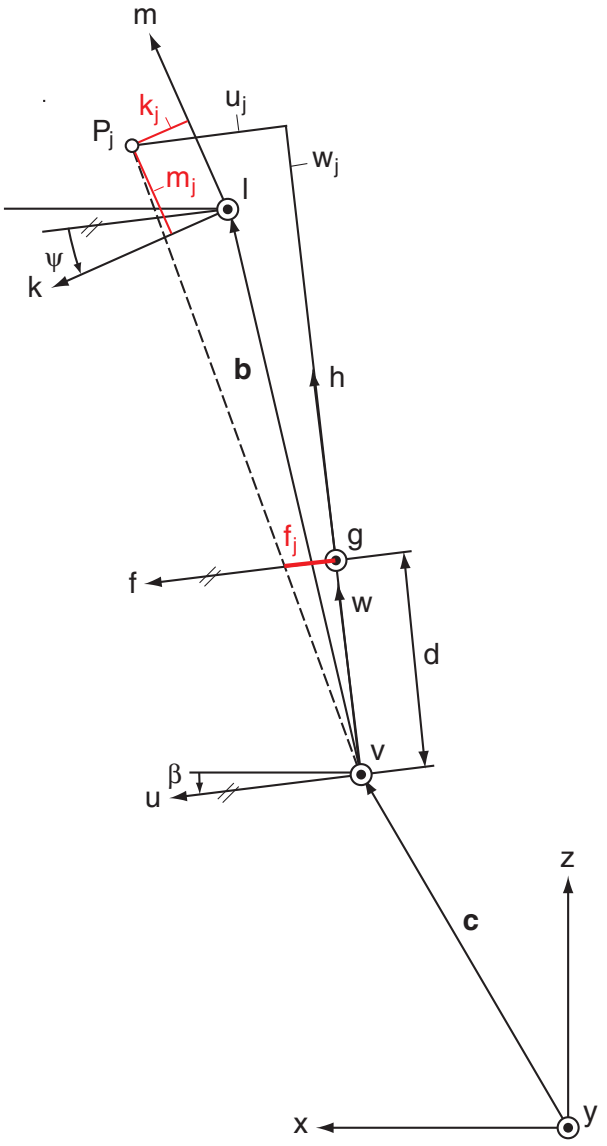


Fig.1 Coordinate systems

3. Kamera

The derivation of the rotation matrices is explained by local coordinates $\mathbf{x}_1$ to $\mathbf{x}_4$.
$\mathbf{x}_1$ is parallel to the base system $\mathbf{x}=(x,y,z)^T$. First rotation by β into $\mathbf{x}_2$, then by α into $\mathbf{x}_3$ and finally by γ into $\mathbf{x}_4$.
The last rotation is not visualized by a mechanical frame. $\mathbf{x}_4$ is identical with $\mathbf{u}=(u,v,w)^T$.

1. Azimuth β about the vertical axis $y_1=y_2$
2. Elevation α about the lateral axis $x_2=x_3$
3. Roll angel γ about the optical axis $z_3=z_4$

A rotation matrix transforms the coordinates of the same point between two coordinate systems. Translations are taken into account by additional terms. A rotation and a translation can be combined in one homogeneous matrix (chapter 6).

$$\mathbf{x}_4 = \mathbf{Z}_{43}\,\mathbf{X}_{32}\,\mathbf{Y}_{21}\,\mathbf{x}_1 = \mathbf{C}_{41}\,\mathbf{x}_1$$

$$\mathbf{Z}_{43} = \begin{bmatrix} \cos\gamma & \sin\gamma & 0 \\ -\sin\gamma & \cos\gamma & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{X}_{32} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & \sin\alpha \\ 0 & -\sin\alpha & \cos\alpha \end{bmatrix}$$

$$\mathbf{Y}_{21} = \begin{bmatrix} \cos\beta & 0 & -\sin\beta \\ 0 & 1 & 0 \\ \sin\beta & 0 & \cos\beta \end{bmatrix}$$

Rotation matrices contain always the well-known structure with two sine and two cosine functions. Where to put the minus sign, can be checked easily, here for $\mathbf{Z}_{43}$:
For small and positive angles γ the following relation is found, which can be interpreted easily for positive values $x_3>0$ and $y_3>0$.

$$x_4 = x_3\cos\gamma + y_3\sin\gamma$$

$$\gamma \to 0$$

$$x_4 \to x_3 + y_3\,\gamma > x_3$$

One can see in Fig.3, x_4 is a little larger than x_3, therefore we have the positive sign. Note that the sign arrangement in the matrix looks different for a rotation about the *second* axis.

The complete rotation matrix $\mathbf{C}_{41}$ is found by concatenating (multiplying) all matrices:

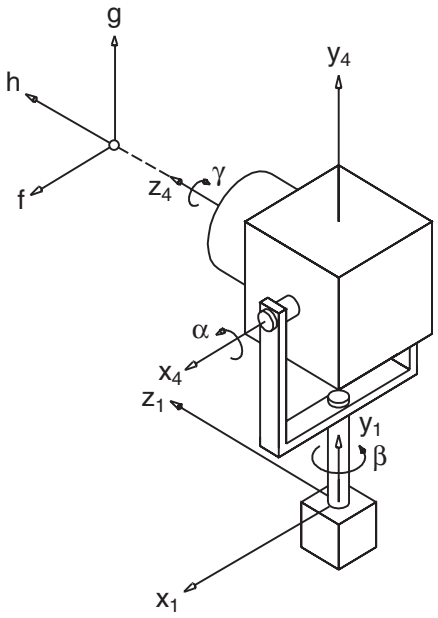


Fig.2 Camera coordinates

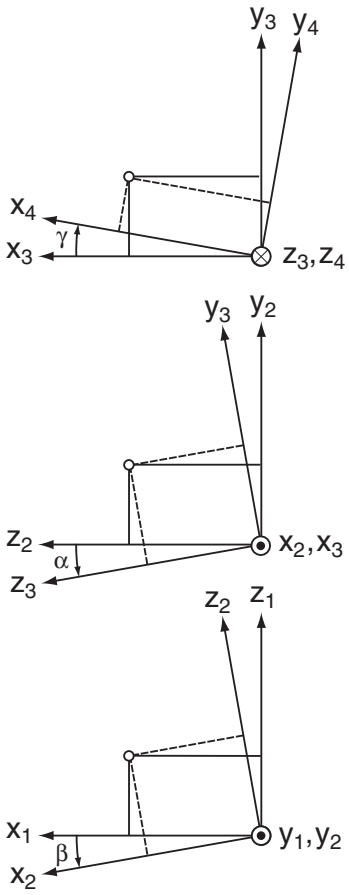


Fig.3 Sequence of camera rotations

$$\mathbf{C}_{41} = \mathbf{Z}_{43}\mathbf{X}_{32}\mathbf{Y}_{21} = \begin{bmatrix} +\cos\beta\cos\gamma + \sin\alpha\sin\beta\sin\gamma & +\cos\alpha\sin\gamma & -\sin\beta\cos\gamma + \sin\alpha\cos\beta\sin\gamma \\ -\cos\beta\sin\gamma + \sin\alpha\sin\beta\cos\gamma & +\cos\alpha\cos\gamma & +\sin\beta\sin\gamma + \sin\alpha\cos\beta\cos\gamma \\ +\cos\alpha\sin\beta & -\sin\alpha & +\cos\alpha\cos\beta \end{bmatrix}$$

4. Object

Again, the derivation of the rotation matrices is explained by local coordinates $\mathbf{x}_1$ to $\mathbf{x}_4$.
$\mathbf{x}_1$ is parallel to the camera system **u**. First rotation by ψ into $\mathbf{x}_2$, then by θ into $\mathbf{x}_3$ and finally by ϕ into $\mathbf{x}_4$.
$\mathbf{x}_4$ is identical with the object or body system $\mathbf{k}=(k,l,m)^T$.

1. Yaw angel ψ about the vertical axis $y_1=y_2$
2. Pitch angle θ about the lateral axis $x_2=x_3$
3. Roll angle ϕ about the long. axis $z_3=z_4$

This sequence of Euler angles is the same as for the camera. It is only necessary to exchange the names of the angles.

$\alpha \rightarrow \theta$

$\beta \rightarrow \psi$

$\gamma \rightarrow \phi$

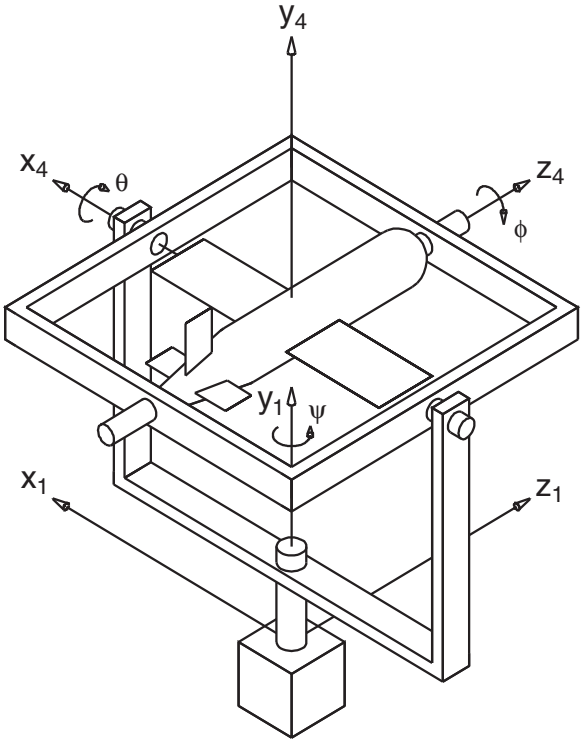


Fig.4 Object coordinates

This final rotation matrix is called **A** because it has been used often for aircraft.

$$\mathbf{A}_{41}=\mathbf{Z}_{43}\mathbf{X}_{32}\mathbf{Y}_{21}=\begin{bmatrix} +\cos\psi\cos\phi+\sin\theta\sin\psi\sin\phi & +\cos\theta\sin\phi & -\sin\psi\cos\phi+\sin\theta\cos\psi\sin\phi \\ -\cos\psi\sin\phi+\sin\theta\sin\psi\cos\phi & +\cos\theta\cos\phi & +\sin\psi\sin\phi+\sin\theta\cos\psi\cos\phi \\ +\cos\theta\sin\psi & -\sin\theta & +\cos\theta\cos\psi \end{bmatrix}$$

5. Coordinate transformations

The same point P with coordinates $\mathbf{k}=(k,l,m)^T$ is represented by coordinate sets in different coordinate systems.
Rotation matrices are orthonormal. The inverse matrix is simply found by transposing. The inverse matrix $\mathbf{A}_{14}=\mathbf{A}_{41}^T$ is named **B** because the object system is sometimes called body system.
A matrix can be replaced by a column of row matrices. For instance, $\mathbf{b}_1^T$ is the first row of matrix **B**. Hopefully this is understandable in the context. Otherwise one could write $\mathbf{b}^1$ for the first row.

Object points **k** are perspectively mapped to image points f,g, using the rule of proportion, see Fig.1 .

$$\mathbf{C}=\mathbf{C}_{14}=\mathbf{C}_{41}^T=\begin{bmatrix}\mathbf{c}_1^T\\ \mathbf{c}_2^T\\ \mathbf{c}_3^T\end{bmatrix}$$

$$\mathbf{B}=\mathbf{A}_{14}=\mathbf{A}_{41}^T=\begin{bmatrix}\mathbf{b}_1^T\\ \mathbf{b}_2^T\\ \mathbf{b}_3^T\end{bmatrix}$$

$$\mathbf{u}=\mathbf{B}\mathbf{k}+\mathbf{b}$$

$$\mathbf{x}=\mathbf{C}\mathbf{u}+\mathbf{c}$$

$$\mathbf{x}=\mathbf{C}(\mathbf{B}\mathbf{k}+\mathbf{b})+\mathbf{c}$$

$$f/d=u/w$$

$$g/d=v/w$$

6. Transformation in homogeneous coordinates

The sequence *first rotation, then translation* can be combined in one matrix, with the special benefit that the perspective transform is contained as well.
The coordinate set is extended by a fourth variable q. After the execution of the matrix multiplication, all variables are divided by the fourth (the first three are divided and the fourth is set to one).
Cascaded transformations, each one possibly a rotation and/or a scaling and/or a translation are ex-executed by matrix multiplication. A pure translation contains the identity matrix **I** instead of a rotation matrix **C**, see below.
It is not necessary to execute the division after translations or rotations, but the division is essential for the perspective transform by means of the matrix $\mathbf{P}_h$.

$$\mathbf{x}_h = \begin{bmatrix} x \\ y \\ z \\ q \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} & c_{13} & c_x \\ c_{21} & c_{22} & c_{23} & c_y \\ c_{31} & c_{32} & c_{33} & c_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u \\ v \\ w \\ 1 \end{bmatrix} = \begin{bmatrix} \mathbf{C} & \mathbf{c} \\ \mathbf{0}^T & 1 \end{bmatrix} \begin{bmatrix} \mathbf{u} \\ 1 \end{bmatrix} = \mathbf{C}_h \, \mathbf{u}_h$$

$$x := x/q; \quad y := y/q; \quad z := z/q$$

$$\mathbf{u}_h = \begin{bmatrix} u \\ v \\ w \\ q \end{bmatrix} = \begin{bmatrix} b_{11} & b_{12} & b_{13} & b_u \\ b_{21} & b_{22} & b_{23} & b_v \\ b_{31} & b_{32} & b_{33} & b_w \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} k \\ l \\ m \\ 1 \end{bmatrix} = \begin{bmatrix} \mathbf{B} & \mathbf{b} \\ \mathbf{0}^T & 1 \end{bmatrix} \begin{bmatrix} \mathbf{k} \\ 1 \end{bmatrix} = \mathbf{B}_h \, \mathbf{k}_h$$

$$u := u/q; \quad v := v/q; \quad w := w/q$$

$$\mathbf{f}_h = \begin{bmatrix} f \\ g \\ h \\ q \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1/d & 0 \end{bmatrix} \begin{bmatrix} u \\ v \\ w \\ 1 \end{bmatrix} = \begin{bmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{0}^T & 1 \end{bmatrix} \begin{bmatrix} \mathbf{u} \\ 1 \end{bmatrix} = \mathbf{P}_h \, \mathbf{u}_h$$

$$f := f/q; \quad g := g/q; \quad h := h/q$$

$$\mathbf{x}_h = \mathbf{C}_h \, \mathbf{u}_h$$
$$\mathbf{u}_h = \mathbf{B}_h \, \mathbf{k}_h$$
$$\mathbf{x}_h = \mathbf{C}_h \, \mathbf{B}_h \, \mathbf{k}_h$$
$$\mathbf{f}_h = \mathbf{P}_h \, \mathbf{B}_h \, \mathbf{k}_h$$

$$f = du/w$$
$$g = dv/w$$

Such a homogeneous operator cannot be inverted by using the inverse matrix. Solving the first equation for **u**, one has to apply first a translation and then a rotation. Each operation can be represented by a homogeneous matrix. Multiplying results in $\mathbf{C}_h^{-1}$ as shown below.

$$\mathbf{x} = \mathbf{C}\,\mathbf{u} + \mathbf{c}$$
$$\mathbf{u} = \mathbf{C}^{-1}(\mathbf{x} - \mathbf{c}) \qquad \text{with} \quad \mathbf{C}^{-1} = \mathbf{C}^T$$

$$\mathbf{u}_h = \begin{bmatrix} \mathbf{C}^{-1} & \mathbf{0} \\ \mathbf{0}^T & 1 \end{bmatrix} \begin{bmatrix} \mathbf{I} & -\mathbf{c} \\ \mathbf{0}^T & 1 \end{bmatrix} \mathbf{x}_h \quad \text{with 3x3 - identity matrix } \mathbf{I}$$

$$\mathbf{u}_h = \begin{bmatrix} \mathbf{C}^{-1} & -\mathbf{C}^{-1}\,\mathbf{c} \\ \mathbf{0}^T & 1 \end{bmatrix} \mathbf{x}_h = \mathbf{C}_h^{-1} \, \mathbf{x}_h$$

7. Nonlinear Collinear Equations and *Newton's Iteration*

The method as described in this chapter is based on reference [2]. An arbitrary point P_j with object coordinates $\mathbf{k}_j=(k,l,m)_j^T$ is perspectively mapped to the image plane as follows. Note: vectors $\mathbf{b}_1^T$ etc. are rows.

$$f_j = d\frac{u_j}{w_j} = d\frac{b_u + \mathbf{b}_1^T\mathbf{k}_j}{b_w + \mathbf{b}_3^T\mathbf{k}_j}$$

$$g_j = d\frac{v_j}{w_j} = d\frac{b_v + \mathbf{b}_2^T\mathbf{k}_j}{b_w + \mathbf{b}_3^T\mathbf{k}_j}$$

For a rectangle $P_1,...,P_4$ one has four pairs of these equations. Unknown are the three translations b_u, b_v, b_w and the three Euler angles ψ,θ,ϕ.

These variables can be combined in one vector of unknowns:

$$\mathbf{p} = (b_u, b_v, b_w, \psi, \theta, \phi)^T$$

Because of the many trigonometric functions in $\mathbf{b}_1^T, \mathbf{b}_2^T, \mathbf{b}_3^T$ (the rows of the rotation matrix **B)**, the system of equations is strongly nonlinear. The division itself is already a nonlinear operation.
A solution is possible by *Newton's Iteration*:

$f_j = d\frac{u_j}{w_j} = d\frac{b_u + \mathbf{b}_1^T\mathbf{k}_j}{b_w + \mathbf{b}_3^T\mathbf{k}_j}$	using u_j, v_j, w_j like here
$g_j = ...$	and in g_j as above delivers
$F_1 = f_1/d - u_1/w_1 = 0$	8 Collinear Equations
...	
$F_4 = f_4/d - u_4/w_4 = 0$	
$F_5 = g_1/d - v_1/w_1 = 0$	
...	
$F_8 = g_4/d - v_4/w_4 = 0$	
$\mathbf{F} = \mathbf{0}$	is expected for the solution $\mathbf{p}$
$\mathbf{p} = \mathbf{p}_i$	start parameters
$\mathbf{F} = \mathbf{F}_i + \mathbf{J}_i\,\mathbf{dp} = \mathbf{0}$	$\mathbf{F}_i = \mathbf{F}(\mathbf{p}_i)$, correction by $\mathbf{dp}$
$\mathbf{J}_i = \begin{bmatrix} \partial F_1/\partial p_1 & ... & \partial F_1/\partial p_6 \\ ... & ... & ... \\ \partial F_8/\partial p_1 & ... & \partial F_8/\partial p_6 \end{bmatrix}_i$	*Jacobi matrix* at $\mathbf{p}_i$
$\mathbf{J}_i\,\mathbf{dp} = -\mathbf{F}_i$	8 equations, 6 unknowns
$\mathbf{J}_i^T\mathbf{J}_i\,\mathbf{dp} = -\mathbf{J}_i^T\mathbf{F}_i$	*Gaussian normal transform*
$\mathbf{M}\,\mathbf{dp} = \mathbf{m}$	6 equations, symmetric matrix
$\mathbf{p}_i = \mathbf{p}_i + \mathbf{dp}$	improved parameter vector

At first one puts up the eight *Collinear Equations* $F_1=0$ ... $F_8=0$ using the complete quotients. The equations are zero for correct parameters **p**. The parameters are changed with *Newton's method*. This requires the so-called *Jacobi matrix* **J** which contains all partial derivatives. These can be approximated by numerical differentiation. This leads to a linearization of the function **F** in a six-dimensional space.
One expects $\mathbf{F}=\mathbf{0}$ for $\mathbf{p}=\mathbf{p}_i+\mathbf{dp}$. The system has 8 linear equations for 6 unknowns. A least squares compromise is found by the so-called *Gaussian normal transform* which delivers 6 linear equations for 6 unknowns. **M** is symmetric, therefore the equations can be solved by the method of *Cholesky* [1] or by the *Gaussian equation solver* [1], as shown in chapter 11. *Cholesky* is somewhat faster.
The process has to be repeated until a norm of **F** is sufficiently small**.** The costs are rather high, especially for the partial derivatives. It would be necessary to represent the trigonometric functions by tables.

Unfortunately it is not guaranteed that *Newton's Iteration* converges. Therefore it is in the moment not recommended to apply this algorithm.

8. Linearized Collinear Equations

The transformation from object to camera coordinates is started by this equation:

$\mathbf{u} = \mathbf{Bk} + \mathbf{b}$

For small angles $\alpha=\psi,\theta,\phi$ one may replace $\cos\alpha=1$, $\sin\alpha=\alpha$ and ignore products $\psi\phi$ etc. Thus, after multiplying with the common denominator, these linear equations are found:

$$\mathbf{u}_j = \hat{\mathbf{B}}\mathbf{k}_j + \mathbf{b}$$

$$\hat{\mathbf{B}} = \begin{bmatrix} 1 & -\phi & +\psi \\ +\phi & 1 & -\theta \\ -\psi & +\theta & 1 \end{bmatrix} = \begin{bmatrix} \hat{\mathbf{b}}_1^T \\ \hat{\mathbf{b}}_2^T \\ \hat{\mathbf{b}}_3^T \end{bmatrix}$$

$$f_j/d = \frac{u_j}{w_j} = \frac{b_u + \hat{\mathbf{b}}_1^T\mathbf{k}_j}{b_w + \hat{\mathbf{b}}_3^T\mathbf{k}_j}$$

$$g_j/d = \frac{v_j}{w_j} = \frac{b_v + \hat{\mathbf{b}}_2^T\mathbf{k}_j}{b_w + \hat{\mathbf{b}}_3^T\mathbf{k}_j}$$

Multiplied by the denominator:

$$(f_j/d)\,(b_w + \hat{\mathbf{b}}_3^T\mathbf{k}_j) = b_u + \hat{\mathbf{b}}_1^T\mathbf{k}_j$$
$$(g_j/d)\,(b_w + \hat{\mathbf{b}}_3^T\mathbf{k}_j) = b_v + \hat{\mathbf{b}}_2^T\mathbf{k}_j$$

Abbreviation:

$$\bar{f}_j = f_j/d$$
$$\bar{g}_j = g_j/d$$

$$\bar{f}_j\,(b_w - \psi k_j + \theta l_j + m_j) = b_u + k_j - \phi l_j + \psi m_j$$
$$\bar{g}_j\,(b_w - \psi k_j + \theta l_j + m_j) = b_v + \phi k_j + l_j - \theta m_j$$

These equations are valid for arbitrary points P_j. They are linear with respect to the unknown parameters $\mathbf{p}=(b_u,b_v,b_w,\psi,\theta,\phi)^T$.

b_u	b_v	b_w	ψ	θ	ϕ	=	RHS
-1	0	$\bar{f}_j$	$-(\bar{f}_j k_j + m_j)$	$\bar{f}_j l_j$	l_j	=	$k_j - \bar{f}_j m_j$
0	-1	$\bar{g}_j$	$-\bar{g}_j k_j$	$(\bar{g}_j l_j + m_j)$	$-k_j$	=	$l_j - \bar{g}_j m_j$

The intended rectangle is defined in the plane k,l , therefore one has generally $m_j=0$:

b_u	b_v	b_w	ψ	θ	ϕ	=	RHS
-1	0	$\bar{f}_j$	$-\bar{f}_j k_j$	$\bar{f}_j l_j$	l_j	=	k_j
0	-1	$\bar{g}_j$	$-\bar{g}_j k_j$	$\bar{g}_j l_j$	$-k_j$	=	l_j

RHS means *right-handed side*

Four points with $m_j = 0$ deliver these eight equations:

b_u	b_v	b_w	ψ	θ	ϕ	=	RHS
–1	0	$\bar{f}_1$	$-\bar{f}_1 k_1$	$\bar{f}_1 l_1$	l_1	=	k_1
–1	0	$\bar{f}_2$	$-\bar{f}_2 k_2$	$\bar{f}_2 l_2$	l_2	=	k_2
–1	0	$\bar{f}_3$	$-\bar{f}_3 k_3$	$\bar{f}_3 l_3$	l_3	=	k_3
–1	0	$\bar{f}_4$	$-\bar{f}_4 k_4$	$\bar{f}_4 l_4$	l_4	=	k_4
0	–1	$\bar{g}_1$	$-\bar{g}_1 k_1$	$\bar{g}_1 l_1$	$-k_1$	=	l_1
0	–1	$\bar{g}_2$	$-\bar{g}_2 k_2$	$\bar{g}_2 l_2$	$-k_2$	=	l_2
0	–1	$\bar{g}_3$	$-\bar{g}_3 k_3$	$\bar{g}_3 l_3$	$-k_3$	=	l_3

The rectangle has edge lengths 2a and 2b with this assignment for the points:

	k_j	l_j
P_1	+a	+b
P_2	–a	+b
P_3	–a	–b

These are the final equations:

b_u	b_v	b_w	ψ	θ	ϕ	=	RHS
–1	0	$\bar{f}_1$	$-a\bar{f}_1$	$+b\bar{f}_1$	+b	=	+a
–1	0	$\bar{f}_2$	$+a\bar{f}_2$	$+b\bar{f}_2$	+b	=	–a
–1	0	$\bar{f}_3$	$+a\bar{f}_3$	$-b\bar{f}_3$	–b	=	–a
–1	0	$\bar{f}_4$	$-a\bar{f}_4$	$-b\bar{f}_4$	–b	=	+a
0	–1	$\bar{g}_1$	$-a\bar{g}_1$	$+b\bar{g}_1$	–a	=	+b
0	–1	$\bar{g}_2$	$+a\bar{g}_2$	$+b\bar{g}_2$	+a	=	+b
0	–1	$\bar{g}_3$	$+a\bar{g}_3$	$-b\bar{g}_3$	+a	=	–b

The matrix **M** and the right-hand side **r** (RHS) are filled according to this scheme. Unknown is the parameter vector **p**. For 8 equations with 6 unknowns the *Gaussian normal transform* by means of matrix $\mathbf{M}^T$ is applied, which delivers a new system with 6 equations. This is actually a least squares approximation for the somewhat contradictory original system. The new system can be solved by *Cholesky* [6] or a by a standard *Gaussian equation solver*, as in chapter [11].

$$\mathbf{p} = (b_u, b_v, b_w, \psi, \theta, \phi)^T$$

$$\mathbf{M}\mathbf{p} = \mathbf{r}$$

$$\mathbf{M}^T\mathbf{M}\,\mathbf{p} = \mathbf{M}^T\mathbf{r}$$

$$\mathbf{N}\mathbf{p} = \mathbf{n}$$

9. Euler angles with respect to the base system.

So far it is possible to calculate the coordinates of object points P in the base system, but the Euler angles ψ',θ',ϕ' of the object with respect to this system are unknown. The information is hidden in matrix **D**.

$$\mathbf{u} = \mathbf{B}\,\mathbf{k} + \mathbf{b}$$
$$\mathbf{x} = \mathbf{C}\,\mathbf{u} + \mathbf{c}$$
$$\mathbf{x} = \mathbf{C}\,(\mathbf{B}\,\mathbf{k} + \mathbf{b}) + \mathbf{c}$$

$$\mathbf{x} = \mathbf{C}\,\mathbf{B}\,\mathbf{k} + \mathbf{C}\,\mathbf{b} + \mathbf{c}$$
$$\mathbf{x} = \mathbf{D}\,\mathbf{k} + \mathbf{...}$$

Matrix **D**=**CB** can be expressed by angles ψ',θ',ϕ' , which are meant as in Fig.4 , but with respect to the base system instead of the camera system. Thus, one simply writes matrix **D** like $\mathbf{A}_{14}$ and call it $\mathbf{A}'_{14}$.

$$\mathbf{A}'_{14} = \mathbf{A}'^{\mathrm{T}}_{41} = \mathbf{D} = \begin{bmatrix} +\cos\psi'\cos\phi' + \sin\theta'\sin\psi'\sin\phi' & -\cos\psi'\sin\phi' + \sin\theta'\sin\psi'\cos\phi' & +\cos\theta'\sin\psi' \\ +\cos\theta'\sin\phi' & +\cos\theta'\cos\phi' & -\sin\theta' \\ -\sin\psi'\cos\phi' + \sin\theta'\cos\psi'\sin\phi' & +\sin\psi'\sin\phi' + \sin\theta'\cos\psi'\cos\phi' & +\cos\theta'\cos\psi' \end{bmatrix}$$

$$\mathbf{D} = (d_{ik})$$

$$\tan\psi' = \frac{d_{13}}{d_{33}}$$

$$\tan\phi' = \frac{d_{21}}{d_{22}}$$

$$\tan\theta' = \frac{(-d_{23})}{\sqrt{d_{21}^2 + d_{22}^2}}$$

The angles are calculated using the well-known four-quadrant arcustangent *atan2*. In the programming language C one gets the angle in radiants in the range $\pm\pi$, but in PostScript in degrees in the range 0° to 360°.

angle = *atan2* (numerator, denominator)

The whole calculation is obsolete if the camera is not rotated, **C**=**I** .

10. Results

It is not clear whether the linearized version is suitable for practical applications, for instance for face tracking in front of a monitor.
This has been tested by a PostScript program (chapter 11). Only the transformation between the object system and the camera system is considered. The transformation for the base system is straightforward and therefore it was not tested.
Page0 shows the whole test environment. The image top right is the camera image plane. Cyan dots, connected by a cyan quadrilateral, represent the image of the object rectangle.
Red dots represent the corners if the perspective transform were executed with *calculated* values for the position and the Euler angles.
The numerical values, true and calculated, are shown by numbers, as well are the matrices.
Very important are the so-called residuals and the determinant:

$$\mathbf{R}_M = \mathbf{M}\,\mathbf{p} - \mathbf{r}$$
$$\mathbf{R}_N = \mathbf{N}\,\mathbf{p} - \mathbf{n}$$
$$\mathbf{D} = \det(\mathbf{N})$$

Residuals should be zero. $\mathbf{R}_M$ = resM cannot be zero because eight measured values for six parameters are always to a certain degree contradictory. But this effect is even stronger because of the linearization. $\mathbf{R}_N$ = resN indicates the quality of the numerical solution. The determinant of **N** is as well an important indicator for proper numerical handling.

All geometrical dimensions are given by millimeters. This results in a somewhat unbalanced matrix **M**. The problem is solved by multiplying the edge lengths a,b by a scale factor Sc=0.005. After solving the equations, the components b_u, b_v, b_w have to be divided by the same factor. The choice of the scale factor is not critical. Trial and error, observing all test data on **Page0**, should lead to a reasonable result.
PostScript itself uses in Adobe applications (here InDesign and Acrobat) single precision float, whereas Quite PSAlter [3] seems to work with double precision float, which leads to almost zero residuals $\mathbf{R}_N$.

So far, judging by the images on the next pages, the result looks promising. The linearized equations are probably a good starting point for *Newton's Iteration*.

Even the execution speed is surprisingly high. PostScript is slow because it is an interpreter, opposed to compiling languages. Furtheron, the implementation uses many named variables instead of immediate stack operations.
As a speed test, in procedure *Page0* the procedure *Mapping* was repeated 5000 times. This requires 35 s, which is equivalent to about 140 executions per second. An implementation in C would be much faster.

10.1 Page0

alf bet gam
0.0000
10.0000
0.0000

cx cy cz
15.0000
0.0000
40.0000

d
30.0000

a b
70.0000
120.0000

Sc
0.0050

psi the phi (true) ○ psi the phi (calc)

15.0000	14.1223
0.0000	-0.0000
0.0000	0.0000

bu bv bw (true) ○ bu bv bw (calc)

50.0000	50.7511
0.0000	-0.0000
180.0000	181.2513

M86

-1.0000	0.0000	0.7265	-0.2543	0.4359	0.6000	0.3500
-1.0000	0.0000	-0.0889	-0.0311	-0.0533	0.6000	-0.3500
-1.0000	0.0000	-0.0889	-0.0311	0.0533	-0.6000	-0.3500
-1.0000	0.0000	0.7265	-0.2543	-0.4359	-0.6000	0.3500
0.0000	-1.0000	0.7413	-0.2594	0.4448	-0.3500	0.6000
0.0000	-1.0000	0.6057	0.2120	0.3634	0.3500	0.6000
0.0000	-1.0000	-0.6057	-0.2120	0.3634	0.3500	-0.6000
0.0000	-1.0000	-0.7413	0.2594	0.4448	-0.3500	-0.6000

r81
0.3500
-0.3500
-0.3500
0.3500
0.6000
0.6000
-0.6000
-0.6000

N66

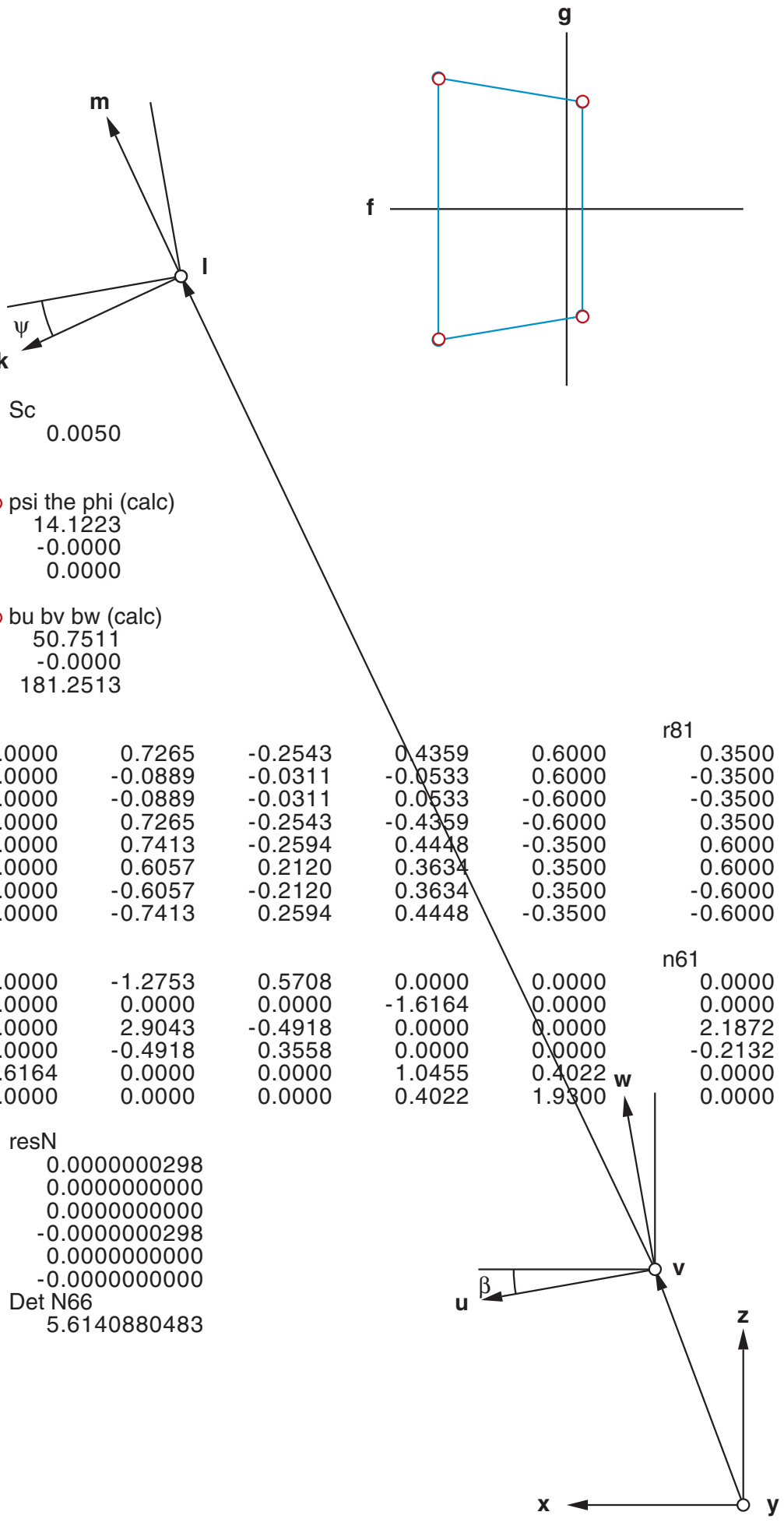
4.0000	0.0000	-1.2753	0.5708	0.0000	0.0000	0.0000
0.0000	4.0000	0.0000	0.0000	-1.6164	0.0000	0.0000
-1.2753	0.0000	2.9043	-0.4918	0.0000	0.0000	2.1872
0.5708	0.0000	-0.4918	0.3558	0.0000	0.0000	-0.2132
0.0000	-1.6164	0.0000	0.0000	1.0455	0.4022	0.0000
0.0000	0.0000	0.0000	0.0000	0.4022	1.9300	0.0000

n61
0.0000
0.0000
2.1872
-0.2132
0.0000
0.0000

resM
-0.0080
0.0080
0.0080
-0.0080
0.0078
0.0012
-0.0012
-0.0078

resN
0.0000000298
0.0000000000
0.0000000000
-0.0000000298
0.0000000000
-0.0000000000

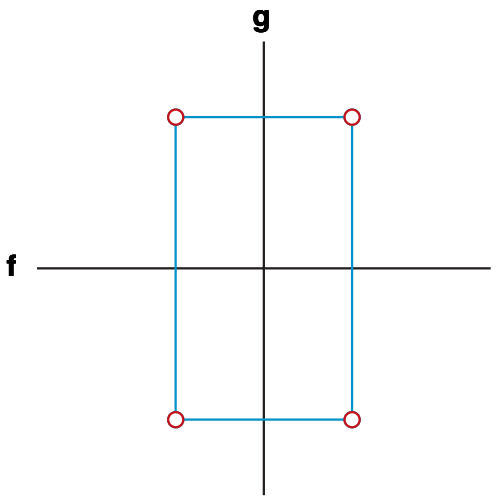
Det N66
5.6140880483



10.2 Page1

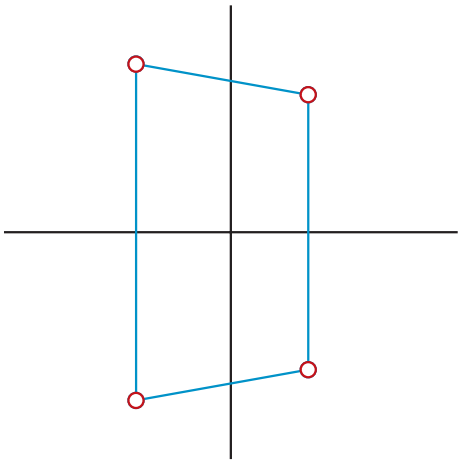
psi the phi (true)	psi the phi (calc)
0.0000	0.0000
0.0000	0.0000
0.0000	0.0000

bu bv bw (true)	bu bv bw (calc)
0.0000	0.0000
0.0000	0.0000
180.0000	180.0000



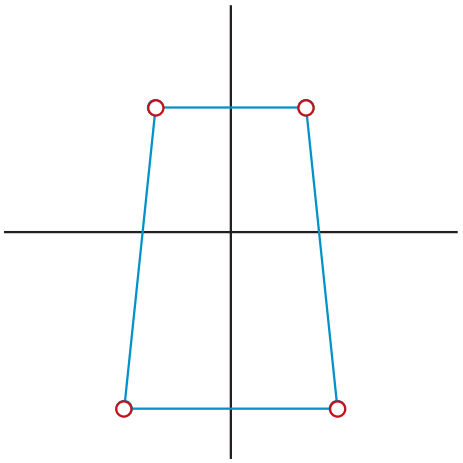
psi the phi (true)	psi the phi (calc)
15.0000	14.9553
0.0000	-0.0000
0.0000	0.0000

bu bv bw (true)	bu bv bw (calc)
0.0000	0.0000
0.0000	-0.0000
180.0000	181.5301



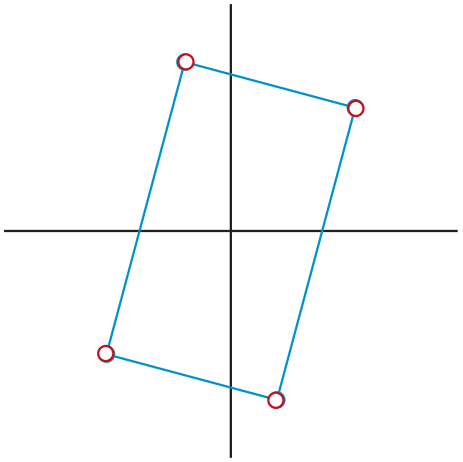
psi the phi (true)	psi the phi (calc)
0.0000	0.0000
15.0000	15.2126
0.0000	0.0000

bu bv bw (true)	bu bv bw (calc)
0.0000	0.0000
0.0000	-0.0000
180.0000	184.6528



psi the phi (true)	psi the phi (calc)
0.0000	0.0000
0.0000	0.0000
15.0000	15.3524

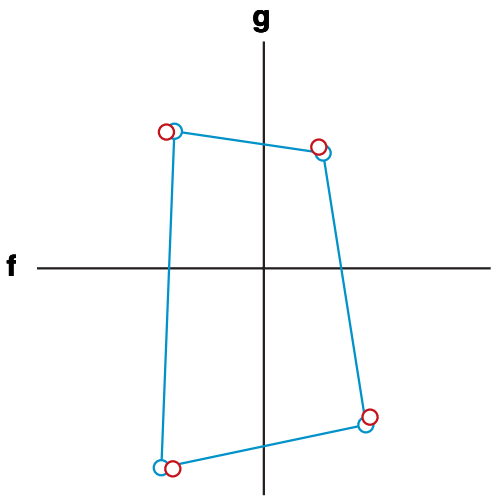
bu bv bw (true)	bu bv bw (calc)
0.0000	-0.0000
0.0000	-0.0000
180.0000	186.3497



10.3 Page2

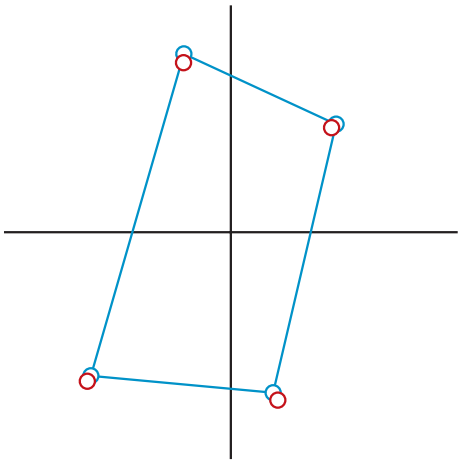
psi the phi (true)	psi the phi (calc)
15.0000	14.3828
15.0000	13.7863
0.0000	-2.9624

bu bv bw (true)	bu bv bw (calc)
0.0000	0.3680
0.0000	-1.4189
180.0000	185.7137



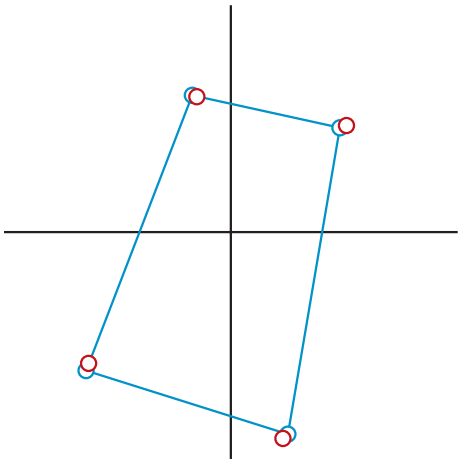
psi the phi (true)	psi the phi (calc)
15.0000	14.8882
0.0000	3.7522
15.0000	15.0888

bu bv bw (true)	bu bv bw (calc)
0.0000	0.1205
0.0000	-0.3389
180.0000	187.8992



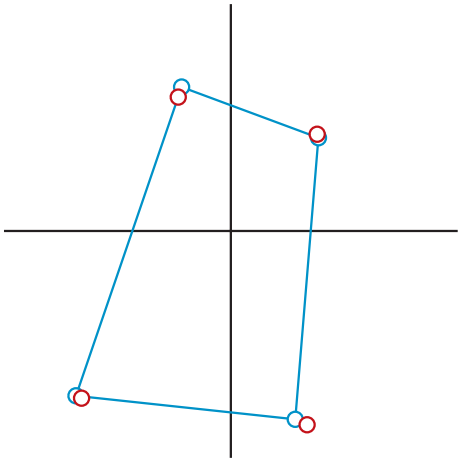
psi the phi (true)	psi the phi (calc)
0.0000	-3.8216
15.0000	15.1428
15.0000	15.6125

bu bv bw (true)	bu bv bw (calc)
0.0000	-0.0950
0.0000	-0.1227
180.0000	191.1313



psi the phi (true)	psi the phi (calc)
15.0000	10.2390
15.0000	18.1650
15.0000	12.3200

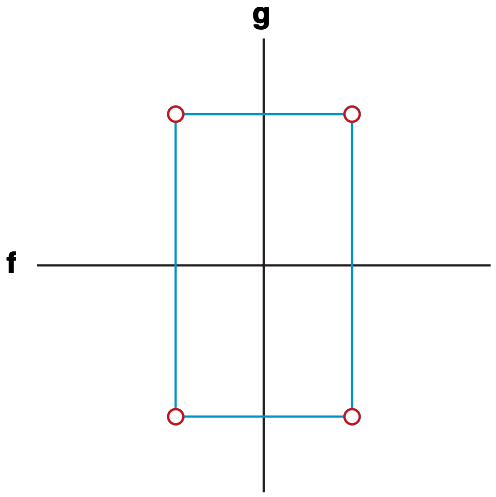
bu bv bw (true)	bu bv bw (calc)
0.0000	0.6935
0.0000	-0.7584
180.0000	191.4585



10.4 Page3

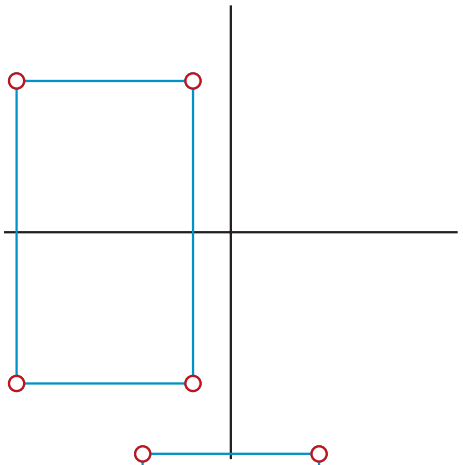
psi the phi (true)	psi the phi (calc)
0.0000	0.0000
0.0000	0.0000
0.0000	0.0000

bu bv bw (true)	bu bv bw (calc)
0.0000	0.0000
0.0000	0.0000
180.0000	180.0000



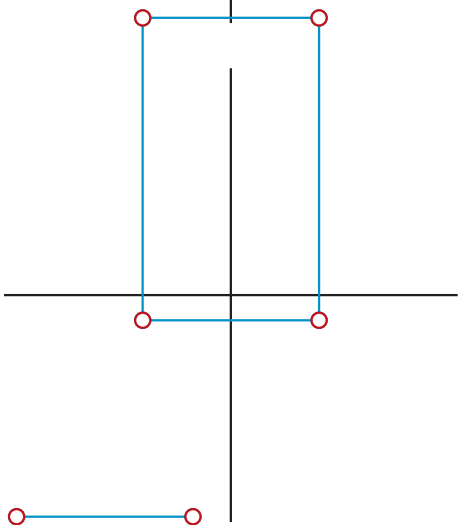
psi the phi (true)	psi the phi (calc)
0.0000	-0.0000
0.0000	0.0000
0.0000	0.0000

bu bv bw (true)	bu bv bw (calc)
100.0000	100.0000
0.0000	0.0000
180.0000	180.0000



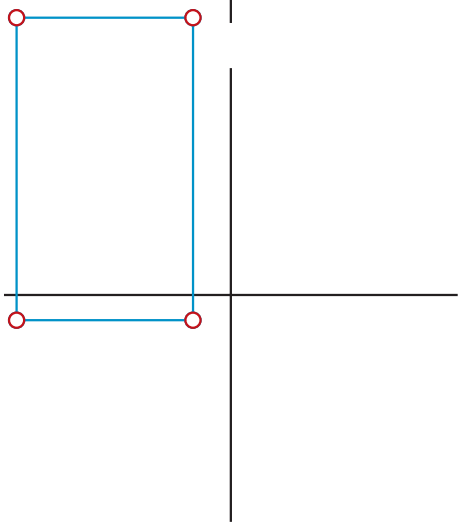
psi the phi (true)	psi the phi (calc)
0.0000	0.0000
0.0000	-0.0000
0.0000	0.0000

bu bv bw (true)	bu bv bw (calc)
0.0000	0.0000
100.0000	100.0000
180.0000	180.0000



psi the phi (true)	psi the phi (calc)
0.0000	-0.0000
0.0000	0.0000
0.0000	-0.0000

bu bv bw (true)	bu bv bw (calc)
100.0000	100.0000
100.0000	100.0000
180.0000	179.9999



11. PostScript program

```
%!PS-Adobe-3.0  EPSF-3.0
%%BoundingBox:  0 0 595 842
%%Creator:      Gernot Hoffmann
%%Title:        PS-1Kamera
%%CreationDate: May 30 / 2012
% Object tracking

/mm {2.834646 mul} def          % convert mm into PostScript units
/mi {2.834646 div} def          % inverse function
/gr {180 mul 3.141592 div} def  % convert rad into degree

/Sc1 [0.005] def                % Scaling for numerical improvement

% Graphics coordinates in mm
/xgrf1 180 def
/ygrf1  20 def
/xgrf2 150 def
/ygrf2 240 def
/dygrf  65 def
/xtxt1  30 def
/xtxt2  64 def
/xtxt3 175 def
/ytxt1 255 def
/ytxt2 100 def
/ax     30 def
/r1      1 def
/r2      1 def
/col1 {1 0 0 0.2 setcmykcolor} def
/col2 {0 1 1 0.2 setcmykcolor} def

% Camera, angles in degree, coordinates in mm
/alf   0 def
/bet  10 def
/gam   0 def
/cx   15 def
/cy    0 def
/cz   40 def
/d    30 def

% Object, angles in degree, coordinates in mm
/psi  15 def
/the   0 def
/phi  10 def
/bu   50 def
/bv    0 def
/bw  180 def

/apo  70 def % Edge lengths a,b
/bpo 120 def
```

```
/SetHelv
{0 0 0 1 setcmykcolor
/fh 11 mi def
/Helvetica findfont fh scalefont setfont
/tms 4 def % after  decimal point
/tmi 4 def % before decimal point (used for space)
} def

/Shownum
% Draw number by string
% Call:     x y nu Shownum
% Requires: font, fh (fontheight), tms (mantissa digits)
% Version May 15 2004 (rounding)
% Version Feb 08 2007 (dict)
%  tx0 ty0 nu :on stack
%  tx0    Position        not overwritten
%  ty0
%  nu     Input number    not overwritten  nu =+-999999
%  fh     Font height     not overwritten
%  tms    Mantissa number of characters    tms=0...6 Global
%  tms=3  Example
%  input  -23.56789  -999.99    0.4567  9999.123456
%  result -23.568    -999.990   0.467   9999.123
%  Postscript number to string is not well defined
%  e.g. 1E-5 instead of 0.00001
%  We use a straightforward BCD conversion.
%  This is always affected by round-off errors
%  because of 32-bit arithmetic
%  Results are different, depending on the interpreter
{ 1 dict
  begin
  /nu  exch def
  /ty0 exch def
  /tx0 exch def
  /tfw fh 0.6 mul def                  % character distance
  /tna nu abs 10 tms neg exp 0.500001 mul add def
  /tdec 1E5 def
  /tchr 1 string def
   tna 999999.1 lt                     % larger number replaced by #
  /tmm true def                        % sign
  {/tx0 tx0 tfw 6 mul sub def
   /tz 0 def
  1 1 5                                % first 5 digits, no leading 0
  { pop
   /tk 0 def
   { tna tdec gt {/tna tna tdec sub def /tk tk 1 add def}{exit} ifelse
   } loop
    tk 0 ne {/tz tz 1 add def} if
    tz 0 ne
    { tx0 ty0 moveto tk tchr cvs show
    } if
    tz 1 eq nu 0 lt and tmm and        % minus  Correction 28/03/2005
    { tx0 tfw 0.7 mul sub ty0 moveto (-) show
     /tmm false def
    } if
   /tdec tdec 0.1 mul def
   /tx0 tx0 tfw add def
```

```
  } for
   /tk 0 def                                % leading 0
   { tna tdec gt {/tna tna tdec sub def /tk tk 1 add def}{exit} ifelse
   } loop
     tmm nu 0 lt and                        % minus
    { tx0 tfw 0.7 mul sub ty0 moveto (-) show
    } if
    tx0 ty0 moveto tk tchr cvs show
   /tdec tdec 0.1 mul def
   /tx0 tx0 tfw add def
  tms 0 gt                                  % for float
  { tx0 ty0 moveto (.) show
   /tx0 tx0 tfw 0.5 mul add def
  1 1 tms
  { pop
   /tk 0 def
   { tna tdec gt {/tna tna tdec sub def /tk tk 1 add def}{exit} ifelse
   } loop
     tx0 ty0 moveto tk tchr cvs show
   /tdec tdec 0.1 mul def
   /tx0 tx0 tfw add def
  } for
  } if
 }{ tx0 tfw sub ty0 moveto (#) show} ifelse
  end
} def

/MatPrn % Print matrix Mnm formatted
% Call:     xtxt ytxt n m (name) Mnm MatPrn
% Requires: font, fh (fontheight),tms (mantissadigits),tmi (leading digits)
{1 dict
 begin
 /Mnm exch def
 /nam exch def
 /m exch def
 /n exch def
 /y exch def
 /x exch def
 /a x def
 /fw fh 0.75 mul def
 /dx tms tmi add fw mul def
 /dy fh def
  x tmi 1 sub fw mul sub y moveto nam show
 /y y dy sub def
 /j 0 def
  n
  {/x a def
    m
    { x y Mnm j get Shownum
     /j j 1 add def
     /x x dx add def
    } repeat
   /y y dy sub def
  } repeat
  end
} def % MatPrn
```

```
/LF % n+2 linefeeds
% Call:     n LF
% Requires: ytxt, fh(fontheight)
{ 2 add {/ytxt ytxt fh sub def} repeat
} def % LF

/GF % n+2 linefeeds, graphic feed
% Call:     n GF
% Requires: ygrf, fh(fontheight)
{ 2 add {/ygrf ygrf fh sub def} repeat
} def % GF

/Bbox % Bounding box A4 portrait
% Call: Bbox
{/bx 595 def
 /by 842 def
 0 0 0 1 setcmykcolor
 1 setlinewidth
 0 0 moveto bx 0 lineto bx by lineto
 0 by lineto closepath stroke
} def % Bbox

/Circle
% Call: x y r Circle
% Uses: global Lw setlinewidth
{1 dict
 begin
  newpath
 /r exch def /y exch def /x exch def
  x y r 0 360 arc
  gsave
  0 0 0 0 setcmykcolor
  fill
  grestore
  % color external
  stroke
  end
} def % Circle

/ArrowTo
% Call: x y ArrowTo
% Uses: global Lw setlinewidth
% To be applied like lineto
% but contains already stroke and fill
{1 dict
 begin
 /y2 exch def
 /x2 exch def
  currentpoint
 /y1 exch def
 /x1 exch def
 /a Lw 12 mul def % length of arrow
 /b Lw 3  mul def % half width
 /dx x2 x1 sub def
 /dy y2 y1 sub def
 /le dx dup mul dy dup mul add sqrt def
 /an dy dx atan def
```

```
  gsave
  x1 y1 translate
  an rotate
  0 0 moveto le a sub 0 lineto stroke
  newpath
  le 0 moveto a neg b rlineto 0 b b add neg
  rlineto closepath
  fill
  grestore
  x2 y2 moveto
 end
} def % ArrowTo

/VecFil % Fill vector by scalars
% Call:   x1 x2 ... xn n vn VecFil
% Requires: none
% vn n array def
{1 dict
 begin
 /vn exch def
 /n  exch def
 /i  n def
  1 1 n
  {/i i 1 sub def
   pop
  /xi exch def
   vn i xi put
  } for
 end
 } def % VecFil

/MatFi1 % --> Anm % Identity matrix, square part
% Call:     n m Mnm MatFi1
% Requires: none
{1 dict
 begin
 /Anm exch def
 /m exch def
 /n exch def
 /j 0 def
  n m mul
  { Anm j 0 put
   /j j 1 add def
  } repeat
 /j 0 def
  n m gt {/n m def} if
  n
  { Anm j 1 put
   /j j m add 1 add def
  } repeat
  end
} def % MatFi1

/MatSca % S1*Anm --> Bnm % Multiply by scalar
% Call:     n m S1 Anm Bnm MatSca
% Requires: none
% S1 1 array def % for printing my MatPrn
```

```
{1 dict
 begin
 /Bnm exch def
 /Anm exch def
 /S1  exch def
 /m exch def
 /n exch def
 /f S1 0 get def
 /j 0 def
  n m mul
  {Bnm j Anm j get f mul put
   /j j 1 add def
  } repeat
  end
} def % MatSca

/MatAdd % Anm+Bnm --> Cnm % Sum
% Call:     n m Anm Bnm Cnm MatAdd
% Requires: none
{1 dict
 begin
 /Cnm exch def
 /Bnm exch def
 /Anm exch def
 /m exch def
 /n exch def
 /j 0 def
  n m mul
  {Cnm j Anm j get Bnm j get add put
   /j j 1 add def
  } repeat
  end
} def % MatAdd

/MatSub % Anm-Bnm --> Cnm % Difference
% Call:     n m Anm Bnm Cnm MatAdd
% Requires: none
{1 dict
 begin
 /Cnm exch def
 /Bnm exch def
 /Anm exch def
 /m exch def
 /n exch def
 /j 0 def
  n m mul
  {Cnm j Anm j get Bnm j get sub put
   /j j 1 add def
  } repeat
  end
} def % MatSub

/MatMul % Anp*Bpm --> Cnm % Product
% Call:     n p m Anp Bpm Cnm MatMul
% Requires: none
{1 dict
 begin
```

```
 /Cnm exch def
 /Bpm exch def
 /Anp exch def
 /m exch def
 /p exch def
 /n exch def
 /ik 0 def
 /i 0 def
  n
  {/k 0 def
  m
  {/s 0 def
   /j 0 def
   p
   {/s s Anp i p mul j add get
         Bpm j m mul k add get mul add def
    /j j 1 add def
   } repeat % p
   Cnm ik s put
  /ik ik 1 add def
  /k k 1 add def
  } repeat % k
  /i i 1 add def
  } repeat % i
 end
} def % MatMul

/MatTrn % Anm --> Bmn % Transposed
% Call:     n m Anm Bmn MatTrn
% Requires: none
{1 dict
 begin
 /Bmn exch def
 /Anm exch def
 /m exch def
 /n exch def
 /i 0 def
 m
 {/k 0 def
  n
  {Bmn i n mul k add Anm k m mul i add get put
  /k k 1 add def
  } repeat
 /i i 1 add def
 } repeat
 end
} def % MatTrn

/MatEulC  % Euler angles --> R41
% Call:     alf bet gam C41 MatEulC
% Requires: none
% Rotation matrix for camera angles
% X4 = C41*X1
{1 dict
begin
/C41 exch def
/gam exch def
```

```
/bet exch def
/alf exch def
/C31  9 array def
/calf alf cos def /salf alf sin def /nalf salf neg def
/cbet bet cos def /sbet bet sin def /nbet sbet neg def
/cgam gam cos def /sgam gam sin def /ngam sgam neg def
/Y21 [ cbet 0  nbet
       0    1  0
       sbet 0  cbet] def
/X32 [ 1  0  0
       0  calf salf
       0  nalf calf] def
/Z43 [ cgam sgam 0
       ngam cgam 0
       0    0    1 ] def
3 3 3 X32 Y21 C31 MatMul
3 3 3 Z43 C31 C41 MatMul
end
} def % MatEulC

/MatLin % Ann*Xn=Yn --> Xn and det(Ann)
% Call:     n n Ann Xn Yn D1 MatLin
% Requires: none
% First n is ignored
% D1 1 array def
% Recover Ann and Yn
% Schwarz, Numerische Mathematik, Teubner / 1993
{1 dict
begin
/d1  exch def
/Yn  exch def
/Xn  exch def
/Ann exch def
/n exch def
 pop
/Yc n array def
/Acc n dup mul array def
 Yn  Yc copy
 Ann Acc copy
% Memory map; i,k on stack; top=N*i+k
/mp
{ exch
  n mul add
} bind def
%
/pa n array def
/ca n array def
/n1 n 1 sub def
/dA 1 def
0 1 n1 1 sub
{/k exch def
 /max 0 def
  pa k 0 put
  k 1 n1
  {/i exch def
   /s 0 def
    k 1 n1
```

```
    {/j exch def
     /s s Ann i j mp get abs add def
    } for %j
     /bik Ann i k mp get abs def
     /q bik s div def
      q max gt {/max q def pa k i put } if
  } for %i
    pa k get k ne
    {/dA dA neg def
      0 1 n1
      {/j exch def
       /h Ann k j mp get def
        Ann k j mp Ann pa k get j mp get put
        Ann pa k get j mp h put
      } for %j
    } if
    /dA dA Ann k k mp get mul def
    k 1 add 1 n1
    {/i exch def
     /aik Ann i k mp get def
     /akk Ann k k mp get def
     /bkk akk abs def
      bkk 1 lt
      {/bik aik abs def
      } if
      Ann i k mp aik akk div put
      k 1 add 1 n1
      {/j exch def
        Ann i j mp Ann i j mp get Ann i k mp get
        Ann k j mp get mul sub put
      } for %j
     } for %i
} for %k
/dA dA Ann n1 n1 mp get mul def
0 1 n1 1 sub
{/k exch def
  pa k get k ne
 {/h Yn k get def
   Yn k Yn pa k get get put
   Yn pa k get h put
 } if
} for %k
0 1 n1
{/i exch def
  ca i Yn i get put
  0 1 i 1 sub
  {/j exch def
    ca i ca i get Ann i j mp get ca j get mul sub put
  } for %j
} for %i
n1 -1 0
{/i exch def
 /s ca i get def
  i 1 add 1 n1
  {/k exch def
   /s s Ann i k mp get Yn k get mul sub def
  } for %k
```

```
 /bik Ann i i mp get abs def
  Yn i s Ann i i mp get div put
} for %i
Yn Xn copy
d1 0 dA put  % Determinant is an array with one element
Yc   Yn copy % recovery
Acc Ann copy
end
} bind def % MatLin
```

/DrawGraph

```
{grestore gsave SetHelv
/Lw 0.3 def
 Lw setlinewidth
+1 mm +1 mm scale
 xgrf1 ygrf1 translate
-1 1 scale
 0 0 0 1 setcmykcolor
 0 0 moveto ax 0  ArrowTo
 0 0 moveto 0 ax  ArrowTo
 0 0 moveto cx cz ArrowTo
 0 0 r2 Circle
 cx cz translate
 0 0  moveto 0 ax lineto stroke
 0 0  moveto ax 0 lineto stroke
 bet neg rotate
 0 0 moveto ax 0  ArrowTo
 0 0 moveto 0 ax  ArrowTo
 0 0 moveto bu bw ArrowTo
 0 0 r2 Circle
 0 0 ax 0.8 mul 0 bet arc stroke
 bu bw translate
 0 0 r2 Circle
 0 0 moveto ax 0  lineto stroke
 0 0 moveto 0 ax  lineto stroke
 psi neg rotate
 0 0 moveto ax 0  ArrowTo
 0 0 moveto 0 ax  ArrowTo
 0 0 r2 Circle
 0 0 ax 0.8 mul 0 psi arc stroke
} def % DrawGraph
```

/DrawDots

```
{grestore gsave SetHelv
/Lw 0.3 def
 Lw setlinewidth
+1 mm +1 mm scale
 xgrf ygrf translate
-1 1 scale
 0 0 0 1 setcmykcolor
 ax neg 0 moveto ax 0 lineto stroke
 0 ax neg moveto 0 ax lineto stroke
 col1
 f1 g1 moveto f2 g2 lineto f3 g3 lineto f4 g4 lineto
 closepath stroke
 f1 g1 r1 Circle
 f2 g2 r1 Circle
```

```
 f3 g3 r1 Circle
 f4 g4 r1 Circle
 col2
 f1c g1c r1 Circle
 f2c g2c r1 Circle
 f3c g3c r1 Circle
 f4c g4c r1 Circle
} def % DrawDots

/Initial
{
/cc  3 array def
/bc  3 array def
/ca  3 array def
/ba  3 array def
/bcc 3 array def
/bac 3 array def

/n 8 def
/m 6 def
/M86 n m mul array def
/M68 m n mul array def
/N66 m m mul array def
/r81 n array def
/n61 m array def
/p61 m array def
/D1  1 array def
/y81 n array def
/z81 n array def
/y61 n array def
/z61 n array def

/C41 9 array def /C33 9 array def
/A41 9 array def /B33 9 array def /B33c 9 array def
/k.1 3 array def /k.2 3 array def /k.3 3 array def /k.4 3 array def
/u.1 3 array def /u.2 3 array def /u.3 3 array def /u.4 3 array def

/ane apo neg def
/bne bpo neg def
 apo bpo 0 3 k.1 VecFil
 ane bpo 0 3 k.2 VecFil
 ane bne 0 3 k.3 VecFil
 apo bne 0 3 k.4 VecFil

 alf bet gam 3 ca VecFil
 cx  cy  cz  3 cc VecFil

/xtxt xtxt1 def
/ytxt ytxt1 def
/xgrf xgrf2 def
/ygrf ygrf2 def

 gsave

} def % Initial
```

```
/Mapping
{psi the phi 3 ba VecFil
 bu  bv  bw  3 bc VecFil

 alf bet gam C41 MatEulC
 the psi phi A41 MatEulC

 3 3 C41 C33 MatTrn
 3 3 A41 B33 MatTrn

 3 3 1 B33 k.1 u.1 MatMul
 3 3 1 B33 k.2 u.2 MatMul
 3 3 1 B33 k.3 u.3 MatMul
 3 3 1 B33 k.4 u.4 MatMul

 3 1 u.1 bc u.1 MatAdd
 3 1 u.2 bc u.2 MatAdd
 3 1 u.3 bc u.3 MatAdd
 3 1 u.4 bc u.4 MatAdd

/f1q u.1 0 get u.1 2 get div def
/f2q u.2 0 get u.2 2 get div def
/f3q u.3 0 get u.3 2 get div def
/f4q u.4 0 get u.4 2 get div def
/g1q u.1 1 get u.1 2 get div def
/g2q u.2 1 get u.2 2 get div def
/g3q u.3 1 get u.3 2 get div def
/g4q u.4 1 get u.4 2 get div def

/f1 f1q d mul def
/f2 f2q d mul def
/f3 f3q d mul def
/f4 f4q d mul def
/g1 g1q d mul def
/g2 g2q d mul def
/g3 g3q d mul def
/g4 g4q d mul def

% Scaling for numerical improvement
/Sc Sc1 0 get def
/apos apo Sc mul def
/anes ane Sc mul def
/bpos bpo Sc mul def
/bnes bne Sc mul def

/M86 [-1   0   f1q   anes f1q mul  bpos f1q mul  bpos
      -1   0   f2q   apos f2q mul  bpos f2q mul  bpos
      -1   0   f3q   apos f3q mul  bnes f3q mul  bnes
      -1   0   f4q   anes f4q mul  bnes f4q mul  bnes
       0  -1   g1q   anes g1q mul  bpos g1q mul  anes
       0  -1   g2q   apos g2q mul  bpos g2q mul  apos
       0  -1   g3q   apos g3q mul  bnes g3q mul  apos
       0  -1   g4q   anes g4q mul  bnes g4q mul  anes ] def
```

```
/r81 [ apos
       anes
       anes
       apos
       bpos
       bpos
       bnes
       bnes ] def

 8 6       M86 M68 MatTrn
 6 8 6 M68 M86 N66 MatMul
 6 8 1 M68 r81 n61 MatMul

 6 6 N66 p61 n61 D1 MatLin

/buc p61 0 get def
/bvc p61 1 get def
/bwc p61 2 get def
/psc p61 3 get gr def % rad to grad
/thc p61 4 get gr def
/phc p61 5 get gr def

% Rescaling
/Sc Sc1 0 get
/buc buc Sc div def
/bvc bvc Sc div def
/bwc bwc Sc div def

 buc bvc bwc 3 bcc VecFil
 psc thc phc 3 bac VecFil
} def % Mapping
```

/Reverse

```
{thc psc phc A41 MatEulC
 3 3 A41 B33c MatTrn
 3 3 1 B33c k.1 u.1 MatMul
 3 3 1 B33c k.2 u.2 MatMul
 3 3 1 B33c k.3 u.3 MatMul
 3 3 1 B33c k.4 u.4 MatMul
 3 1 u.1 bc u.1 MatAdd
 3 1 u.2 bc u.2 MatAdd
 3 1 u.3 bc u.3 MatAdd
 3 1 u.4 bc u.4 MatAdd
/f1c u.1 0 get u.1 2 get div d mul def
/f2c u.2 0 get u.2 2 get div d mul def
/f3c u.3 0 get u.3 2 get div d mul def
/f4c u.4 0 get u.4 2 get div d mul def
/g1c u.1 1 get u.1 2 get div d mul def
/g2c u.2 1 get u.2 2 get div d mul def
/g3c u.3 1 get u.3 2 get div d mul def
/g4c u.4 1 get u.4 2 get div d mul def
} def % Reverse
```

```
/Params
{grestore gsave SetHelv
 +1 mm +1 mm scale
 0 0 0 1 setcmykcolor
 xtxt1 ytxt 3 1 (alf bet gam) ca  MatPrn 3 LF
 xtxt1 ytxt 3 1 (cx  cy  cz ) cc  MatPrn 3 LF
/dd [d] def
 xtxt1 ytxt 1 1 (d) dd MatPrn 1 LF
/ab [apo bpo] def
 xtxt1 ytxt 2 1 (a b) ab MatPrn
 xtxt2 ytxt 1 1 (Sc) Sc1 MatPrn 2 LF
} def % Params

/Result
{grestore gsave SetHelv
 +1 mm +1 mm scale
 0 0 0 1 setcmykcolor
 0.3 setlinewidth
/ytxt0 ytxt def
 col1
 xtxt1 11 sub ytxt 2.1 add 0.3 -33 rectfill
 0 0 0 1 setcmykcolor
 xtxt1 ytxt 3 1 (psi the phi (true)) ba  MatPrn 3 LF
 0 0 0 1 setcmykcolor
 xtxt1 ytxt 3 1 (bu  bv  bw (true)) bc  MatPrn 3 LF
/ytxt ytxt0 def
 col2
 newpath xtxt2 11 sub ytxt 1 add 1 0 360 arc stroke
 0 0 0 1 setcmykcolor
 xtxt2 ytxt 3 1 (psi the phi (calc)) bac MatPrn 3 LF
 0 0 0 1 setcmykcolor
 xtxt2 ytxt 3 1 (bu bv bw (calc))    bcc MatPrn
 col2
 newpath xtxt2 11 sub ytxt 1 add 1 0 360 arc stroke 3 LF
} def % Result

/Checks
{grestore gsave SetHelv
 +1 mm +1 mm scale
 0 0 0 1 setcmykcolor
 xtxt1 ytxt 8 6 (M86) M86 MatPrn
 xtxt3 ytxt 8 1 (r81) r81 MatPrn 8 LF
 xtxt1 ytxt 6 6 (N66) N66 MatPrn
 xtxt3 ytxt 6 1 (n61) n61 MatPrn 6 LF
 8 6 1 M86 p61 y81 MatMul
 8 1   y81 r81 z81 MatSub
 xtxt1 ytxt 8 1 (resM) z81 MatPrn
 6 6 1 N66 p61 y61 MatMul
 6 1   y61 n61 z61 MatSub
/tms 10 def
 xtxt2 ytxt 6 1 (resN) z61 MatPrn 5 LF
 xtxt2 ytxt 1 1 (Det N66) D1 MatPrn 1 LF
} def % Checks

/Make1
{Mapping Result Reverse DrawDots 5 LF 15 GF Text2
} def % Make1
```

```
/Text1
{grestore gsave SetHelv
 0 0 0 1 setcmykcolor
 1 mm 1 mm scale
/fh 11 mi def
/Helvetica-Bold findfont fh scalefont setfont
 145  19 moveto (x) show
 184  19 moveto (y) show
 179  51 moveto (z) show
 131  53 moveto (u) show
 168  59 moveto (v) show
 158  91 moveto (w) show
  53 213 moveto (k) show
  88 229 moveto (l) show
  69 257 moveto (m) show
/Symbol findfont fh scalefont setfont
 135  56 moveto (b) show
  56 219 moveto (y) show
 } def

/Text2
{grestore gsave SetHelv
 1 mm 1 mm scale
/fh 11 mi def
/Helvetica-Bold findfont fh scalefont setfont
 116   239 moveto (f) show
 148.5 272 moveto (g) show
} def

/Page0
{%Bbox
/psi  15 def /the   0 def /phi   0 def
/bu   50 def /bv    0 def /bw  180 def
 Initial
 Params DrawGraph Mapping Result Checks Reverse DrawDots
 Text1 Text2
} def % Page0

/Page1
{Initial
/psi   0 def /the   0 def /phi   0 def
/bu    0 def /bv    0 def /bw  180 def
 Make1
/psi  15 def /the   0 def /phi   0 def
/bu    0 def /bv    0 def /bw  180 def
 Make1
/psi   0 def /the  15 def /phi   0 def
/bu    0 def /bv    0 def /bw  180 def
 Make1
/psi   0 def /the   0 def /phi  15 def
/bu    0 def /bv    0 def /bw  180 def
 Make1
} def % Page1
```

```
/Page2
{Initial
/psi  15 def /the  15 def /phi   0 def
/bu    0 def /bv    0 def /bw  180 def
 Make1
/psi  15 def /the   0 def /phi  15 def
/bu    0 def /bv    0 def /bw  180 def
 Make1
/psi   0 def /the  15 def /phi  15 def
/bu    0 def /bv    0 def /bw  180 def
 Make1
/psi  15 def /the  15 def /phi  15 def
/bu    0 def /bv    0 def /bw  180 def
 Make1
} def % Page2

/Page3
{Initial
/psi   0 def /the   0 def /phi   0 def
/bu    0 def /bv    0 def /bw  180 def
 Make1
/psi   0 def /the   0 def /phi   0 def
/bu  100 def /bv    0 def /bw  180 def
 Make1
/psi   0 def /the   0 def /phi   0 def
/bu    0 def /bv  100 def /bw  180 def
 Make1
/psi   0 def /the   0 def /phi   0 def
/bu  100 def /bv  100 def /bw  180 def
 Make1
} def % Page3

% Choose one
/sel 0 def

sel 0 eq {Page0} if
sel 1 eq {Page1} if
sel 2 eq {Page2} if
sel 3 eq {Page3} if

showpage
```

12. References

[1] H.R.Schwarz
Numerische Mathematik, B.G.Teubner, Stuttgart, 1993

[2] R.C.Hughes
Enhanced single camera photogrammetry algorithms for real time control applications
http://www.isprs.org/proceedings/XXVI/part5/391_XXVI-part5.pdf

[3] PostScript interpreter, editor and debugger PSAlter
http://www.quite.com/psalter/

[4] Teaching a robot by showing the motion
http://docs-hoffmann.de/icip01.pdf

[5] Euler angles, rotation matrices and projections
http://docs-hoffmann.de/euler26112001.pdf

[6] Planar projections
http://docs-hoffmann.de/project18032004.pdf

[7] Illustrations for a lecture Computer Vision
http://docs-hoffmann.de/covi-2005-1-4.pdf

[8] PostScript Tutor with references
http://docs-hoffmann.de/pstutor22112002.pdf

[9] All documents by G.Hoffmann
http://docs-hoffmann.de/howww41a.html

This document:

http://docs-hoffmann.de/track1cam15062012.pdf

Gernot Hoffmann

July 01 / 2012 - February 06 / 2013
Website
Load Browser / Click here