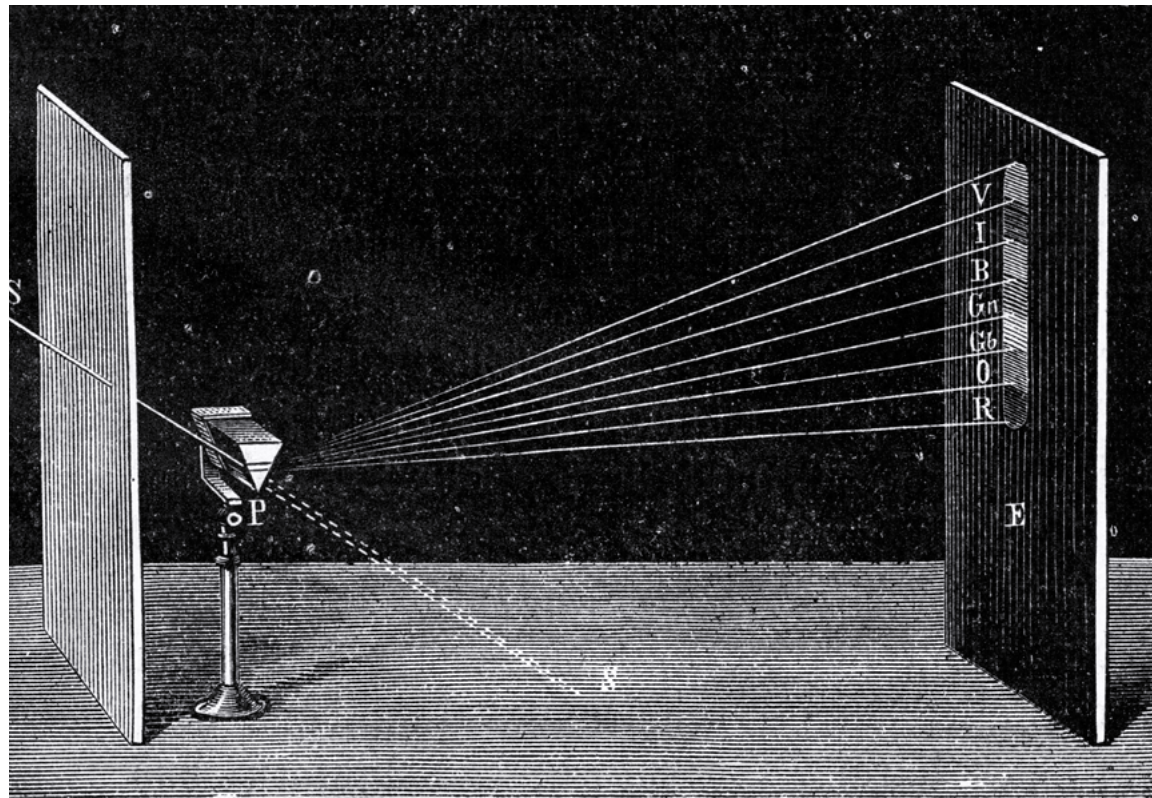


Gernot Hoffmann

Newton's Prism Experiment and Goethe's Objections



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1.1 Introduction / How to use this doc

This document can be read by Adobe Acrobat. Some settings are essential: the color space sRGB for the appearance; the resolution 72 dpi for pixel synchronized images. Best view zoom 200% (title image and photo on p.20).

Settings for Acrobat

Edit / Preferences / General / Page Display (since version 6)
Custom Resolution 72 dpi

Edit / Preferences / General / Color Management (full version only)
sRGB
Euroscale Coated or ISO Coated or SWOP
Gray Gamma 2.2

A reasonable reproduction of the colors is possible only by a calibrated cathode ray tube monitor or a high end LCD monitor (no hope for notebooks or laptops). The monitor should be calibrated or adjusted by these test patterns:

CalTutor

<http://docs-hoffmann.de/caltutor270900.pdf>

Printing this doc accurately requires a calibrated printer, preferably an inkjet or high end color laser printer. Office printers cannot reproduce the spectrum bars reasonably.

Accurate printing requires a calibrated printer

This document is protected by copyright. Any reproduction requires a permission by the author. This should be considered as a protection against bad or wrong copying. The author provides on demand files in RGB for the specific purpose in original quality and files for CMYK offset printing (which cannot reproduce sRGB colors correctly), adjusted as good as possible.

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Gernot Hoffmann

August 2005 and later

1.2 Introduction / About Newton

Isaac Newton lived from 1642 to 1727. His work about optics and color was probably inspired by *Descartes*, *Charleton*, *Boyle* and *Hook* [5]. Refraction and dispersion by prisms were already known at this time, but Newton introduced 1666 a set of three *crucial experiments* (in fact by numerous experiments):

1. Refraction and dispersion of a thin ray, which produces a spectrum band with all rainbow colors: red, orange, yellow, green, blue, indigo and violet.
2. Condensation of the spectrum band by a lens. This delivers again white light.
3. A spectral color, here a small part of the spectrum band, can be refracted again but it cannot be further dispersed.

Indigo was added - though not visible - in order to get a 'harmonic' sequence like the seven tones c,d,e,f,g,a,h,(c). The last tone c has double the frequency of the first. It is a strange coincidence that this ratio is approximately valid for the frequencies or wavelengths of visible light. In computer graphics we have a strong cyan between green and blue. In prism colors it is sometimes detectable - a better candidate than indigo for the seventh rainbow color.

Newton's theories were by no means always clear and understandable, which lead to lengthy and often rather impolite discussions between the most famous scientists in Europe.

It is safe to say: Newton had discovered that white light consists of colored light.

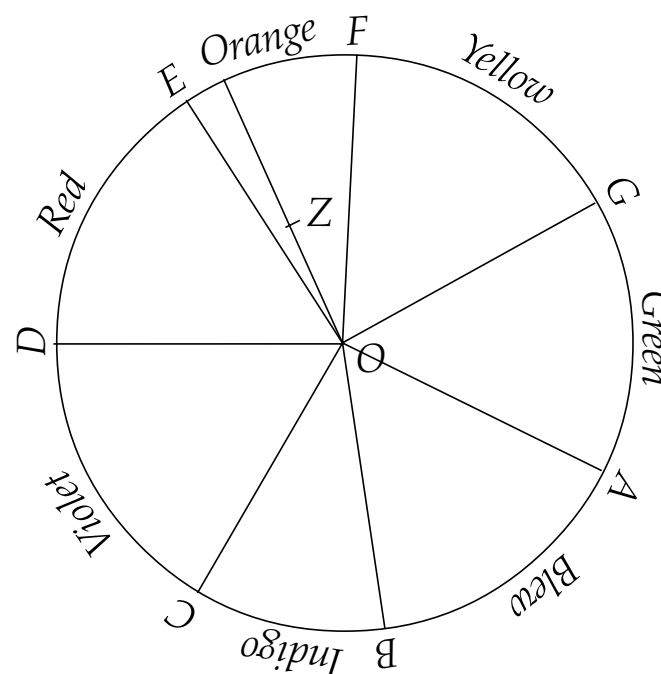
Newton arranged the spectrum on a circle. Drawing by the author, based on a print in *Opticks* [1].

Connecting the ends of a spectrum like this does not have an equivalent for general electromagnetic or acoustical waves.

This discovery was the first step towards spherical or cylindrical three-dimensional color spaces [7], [24].

A segment of the circle is occupied by violet. Newton's violet is complementary to yellow-green.

In our present understanding this is mainly the magenta segment, but magenta is mixed by red and blue (violet) and not a spectral color.



Newton says: 'That if the point Z fall in or near to the line OD, the main ingredients being the red and violet, the Colour compounded shall not be any of the prismatic Colours, but a purple, inclining to red or violet, accordingly as the point Z lieth on the side of the line DO towards E or towards C, and in general the compounded violet is more bright and more fiery than the uncompounded'.

Newton knew already additive color mixing. He applied the *center of gravity rule* (which is accurately valid in the CIE chromaticity diagram) in his wheel. Z is such a mixture.

The different refraction indices for different wavelengths lead Newton to the conclusion that telescopes with mirrors instead of lens' should be more accurate. He manufactured such a telescope, using a metallic spherical mirror. The accuracy was probably affected by spherical aberration. At that time it was assumed that an ideal mirror should have the shape of a cone section but it was not yet discovered which one - a parabola. Manufacturing such a mirror would have been impossible.

Newton published 'Opticks' 1704, nearly forty years after his first experiments. In the time between he had spent much time developing mathematical methods, geometry and principles of mechanics.

1.3 Introduction / About Goethe

Johann Wolfgang von Goethe lived from 1749 to 1832. Well known as poet and writer, he was familiar as well with philosophy and natural science.

1810 he published his book *Geschichte der Farbenlehre* [2]. This is mainly a historical review but also a furious attack against Newton's theories. Judging by our actual understanding of natural science, Goethe's color theory was by no means well founded and elaborated. On the other hand he tried to integrate the aspects of culture and art in his generalizing concepts.

The question remains: why did Goethe not believe in the *Obvious* in Newton's theories ?

It is the purpose of this document to demonstrate that the obvious in our present understanding was by no means obvious for Goethe. This demonstration will be done in an engineering style, based on many illustrations. These are accurate in the sense of classical optics and in the sense of actual color theory.

The author found during his Google search nearly nowhere correct illustrations for the prism experiment. Drawings in publications are often wrong [5],[6], this could be a long list...

One of the best illustrations is the title graphic, dated 1912 [8], but even here rules fantasy over reality: the blue-ish part is too small, indigo is in reality not perceivable and the geometrical path of the red ray is not accurate.

1.4 Introduction / Goethe says...

Some statements by Goethe, important in our context, may be quoted [2], followed by a simplified translation.

A [2, *Zweiter Teil, Konfession des Verfassers*]

Aber wie verwundert war ich, als die durchs Prisma angeschaute weiße Wand nach wie vor weiß blieb, daß nur da, wo ein Dunkles dran stieß, sich eine mehr oder weniger entschiedene Farbe zeigte, daß zuletzt die Fensterstäbe am allerlebhaftesten farbig erschienen, indessen am lichtgrauen Himmel draußen keine Spur von Färbung zu sehen war. Es bedurfte keiner langen Überlegung, so erkannte ich, daß eine Grenze notwendig sei, um Farben hervorzu- bringen, und ich sprach wie durch einen Instinkt sogleich vor mich laut aus, daß die Newto- nische Lehre falsch sei.'

'How surprised was I when I looked through a prism onto a white wall - still white, and colors appeared only there where dark met white. It was immediately clear that colors can appear only at boundaries. And I said loud, guided by an instinct: Newton's teaching is wrong.'

B [2, *Zweiter Teil, Polemischer Teil*]

'Also, um beim Refraktionsfalle zu verweilen, auf welchem sich die Newtonische Theorie doch eigentlich gründet, so ist es keineswegs die Brechung allein, welche die Farbenerscheinung verursacht; vielmehr bleibt eine zweite Bedingung unerläßlich, daß nämlich die Brechung auf ein Bild wirke und ein solches von der Stelle wegrücke.

Ein Bild entsteht nur durch seine Grenzen; und diese Grenzen übersieht Newton ganz, ja er leugnet ihren Einfluß...und keines Bildes Mitte wird farbig, als insofern die farbigen Ränder sich berühren oder übergreifen.'

'Again about the refraction, which is the true basis of Newton's theory: color effects are not generated by refraction alone. A second necessary condition is, that refraction acts on an image by shifting it. An image is created merely by its boundaries, and these boundaries are not taken into account by Newton... and no image will become colorful in the middle, unless the colored boundaries touch or overlap each other.'

C [2, *Zweiter Teil, Beiträge zur Optik*]

'Unter den eigentlichen farbigen Erscheinungen sind nur zwei, die uns einen ganz reinen Begriff geben, nämlich Gelb und Blau. Sie haben die besondere Eigenschaft, daß sie zusam- men eine dritte Farbe hervorbringen, die wir Grün nennen.

Dagegen kennen wir die rote Farbe nie in einem ganz reinen Zustande: denn wir finden, daß sie sich entweder zum Gelben oder zum Blauen hinneigt.'

'Only two colors can be considered as pure: yellow and blue. Their special feature is: they generate together a new color which we call green. Red is never pure: we find that this color tends either to yellow or to blue.'

2. Snell's law and Sellmeier's equation

The refraction of a ray which enters from a medium with refraction index n_1 into a medium with refraction index n_2 is defined by *Snell's law*. The angles α and β are shown on the next page.

The refraction index for air is practically $n_1=1.0$ and with $n_2=n$ we get:

$$\frac{\sin(\beta)}{\sin(\alpha)} = \frac{n_1}{n_2} = \frac{1}{n}$$

Goethe said: *Willebrord Snellius* (1591–1626) knew the fundamentals but he did not know the definition by sine functions [2, *Erster Teil, Fünfte Abteilung*].

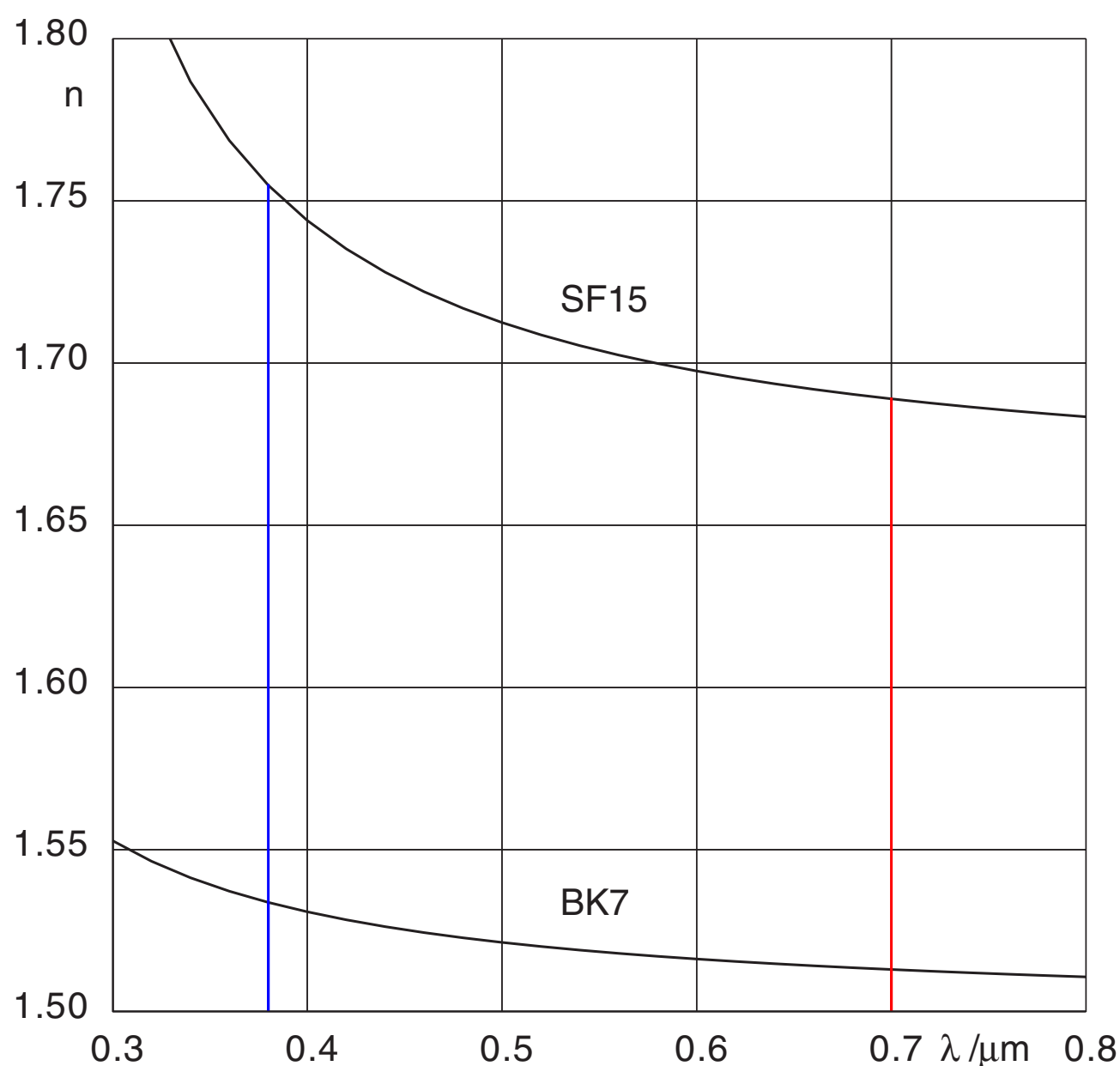
The refraction index can be modelled as a function of the wavelength by *Sellmeier's equation*:

$$n^2(\lambda) = 1 + \frac{B_1 \lambda^2}{\lambda^2 - C_1} + \frac{B_2 \lambda^2}{\lambda^2 - C_2} + \frac{B_3 \lambda^2}{\lambda^2 - C_3}$$

The structure of this function is essentially based on *Maxwell's* theory of continua [4]. The function has three poles which are outside the spectrum of visible light. The poles indicate resonance phenomena without damping. In reality the peaks are finite because of damping. The actual parameters are found by measurements, here for special types of flint and crown glass.

The refraction of flint glass is stronger. Because of the steeper slope the dispersion is stronger as well. Combinations of flint glass and crown glass can be used for achromatic lens or prism systems which show no dispersion for two distinct wavelengths and nearly no dispersions for the others. Goethe knew this already.

Tables are supplied by *Schott AG* [15], here used for flint glass SF15. Thanks to *Danny Rich*. The Sellmeier equation and the data for crown glass BK7 were found at *Wikipedia* [16].

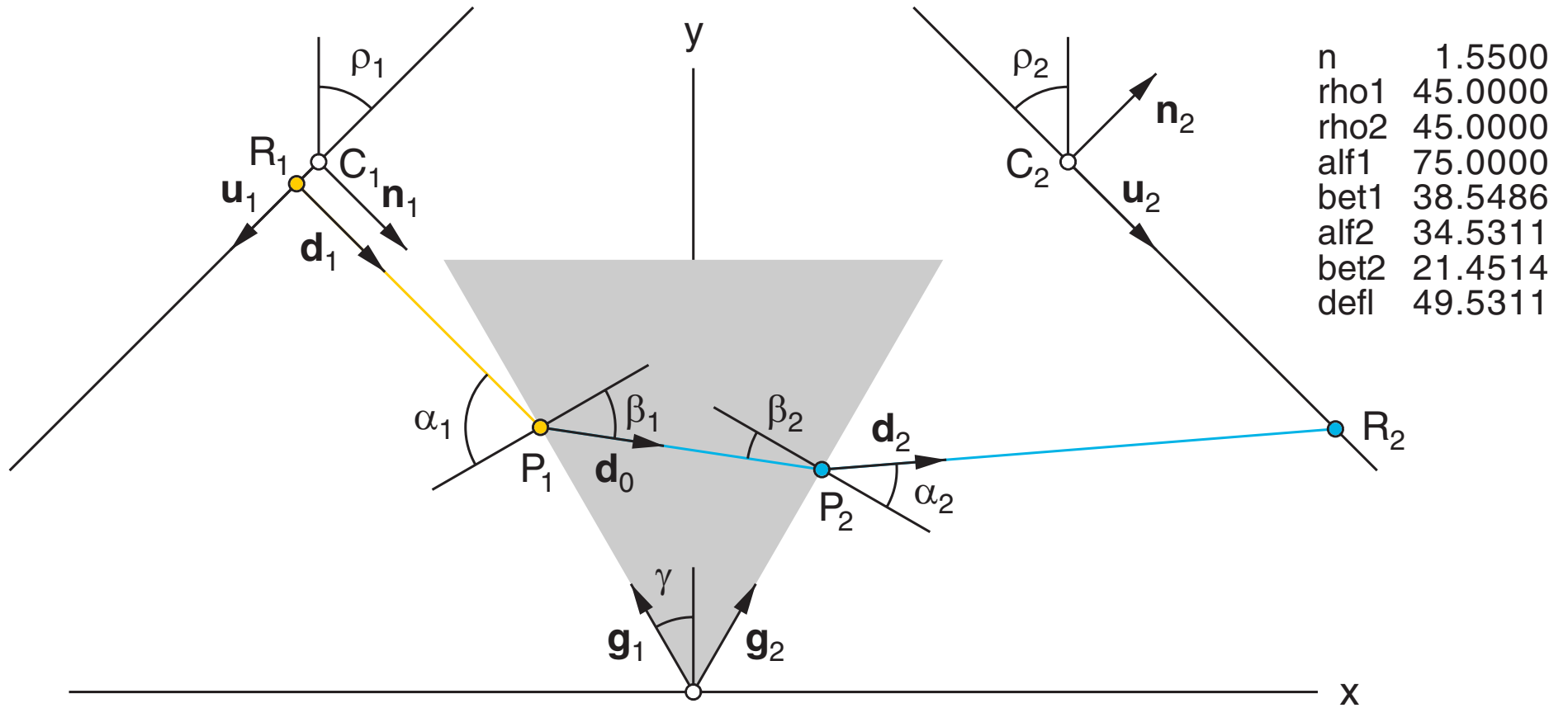


Glass SF15	
B1	1.539259
B2	0.247621
B3	1.038164
C1	0.011931
C2	0.055608
C3	116.416755

Glass BK7	
B1	1.039612
B2	0.231792
B3	1.010469
C1	0.006001
C2	0.020018
C3	103.560653

3. Calculations for the ray path

The path of the ray is calculated by intersections of line segments, using Snell's law and some simple geometry: lines in parametric representation; solutions by Cramer's rule. All direction vectors are normalized for unit length. The origin of \mathbf{u}_1 is at C_1 .



$$\mathbf{g}_1 = \begin{bmatrix} -\sin(\gamma) \\ +\cos(\gamma) \end{bmatrix}$$

$$\mathbf{u}_1 = \begin{bmatrix} -\sin(\rho_1) \\ -\cos(\rho_1) \end{bmatrix}$$

$$\mathbf{d}_1 = \begin{bmatrix} +\cos(\rho_1) \\ -\sin(\rho_1) \end{bmatrix}$$

$$\mathbf{r}_1 = \mathbf{c}_1 + \mathbf{u}_1$$

$$\mathbf{x}_1 = \mathbf{r}_1 + \lambda \mathbf{d}_1$$

$$\mathbf{x}_2 = \mu \mathbf{g}_1$$

$$\mathbf{x}_1 = \mathbf{x}_2$$

$$\mu \mathbf{g}_1 - \lambda \mathbf{d}_1 = \mathbf{r}_1$$

$$\mu g_{1x} - \lambda d_{1x} = r_{1x}$$

$$\mu g_{1y} - \lambda d_{1y} = r_{1y}$$

Solution by Cramer's rule

$$dA = d_{1x} g_{1y} - d_{1y} g_{1x}$$

$$dM = d_{1x} r_{1y} - d_{1y} r_{1x}$$

$$\mu = dM / dA$$

$$\mathbf{p}_1 = \mu \mathbf{g}_1$$

$$\alpha_1 = \gamma + \rho_1$$

Refraction by Snell's law

$$\sin(\beta_1) = (1/n) \sin(\alpha_1)$$

$$\cos(\beta_1) = \sqrt{1 - \sin^2(\beta_1)}$$

$$\beta_1 = \text{atan2}[\sin(\beta_1), \cos(\beta_1)]$$

$$\delta_1 = \gamma - \beta_1$$

$$\mathbf{g}_2 = \begin{bmatrix} +\sin(\gamma) \\ +\cos(\gamma) \end{bmatrix}$$

$$\mathbf{d}_0 = \begin{bmatrix} +\cos(\delta_1) \\ +\sin(\delta_1) \end{bmatrix}$$

$$\mathbf{x}_1 = \mathbf{p}_1 + \lambda \mathbf{d}_0$$

$$\mathbf{x}_2 = \mu \mathbf{g}_2$$

$$dA = d_{0x} g_{2y} - d_{0y} g_{2x}$$

$$dM = d_{0x} p_{1y} - d_{0y} p_{1x}$$

$$\mathbf{p}_2 = (dM / dA) \mathbf{g}_2$$

$$\beta_2 = \gamma + \delta$$

$$\sin(\alpha_2) = n \sin(\beta_2)$$

$$\cos(\alpha_2) = \sqrt{1 - \sin^2(\alpha_2)}$$

$$\alpha_2 = \text{atan2}[\sin(\alpha_2), \cos(\alpha_2)]$$

$$\mathbf{u}_2 = \begin{bmatrix} +\sin(\rho_2) \\ -\cos(\rho_2) \end{bmatrix}$$

$$\delta_2 = \alpha_2 - \gamma$$

$$\mathbf{d}_2 = \begin{bmatrix} +\cos(\delta_2) \\ +\sin(\delta_2) \end{bmatrix}$$

$$\mathbf{x}_1 = \mathbf{p}_2 + \lambda \mathbf{d}_2$$

$$\mathbf{x}_2 = \mathbf{c}_2 + \mu \mathbf{u}_2$$

$$\mathbf{s}_2 = \mathbf{p}_2 - \mathbf{c}_2$$

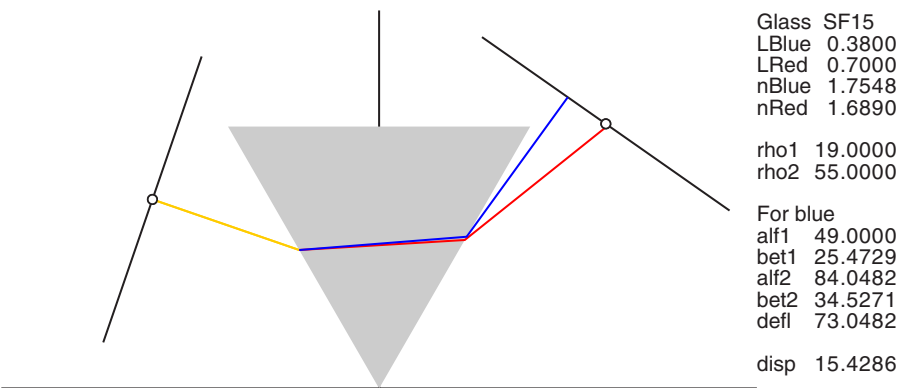
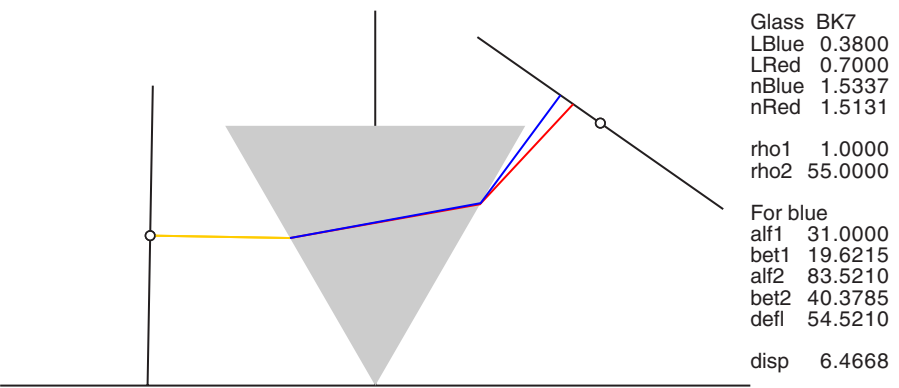
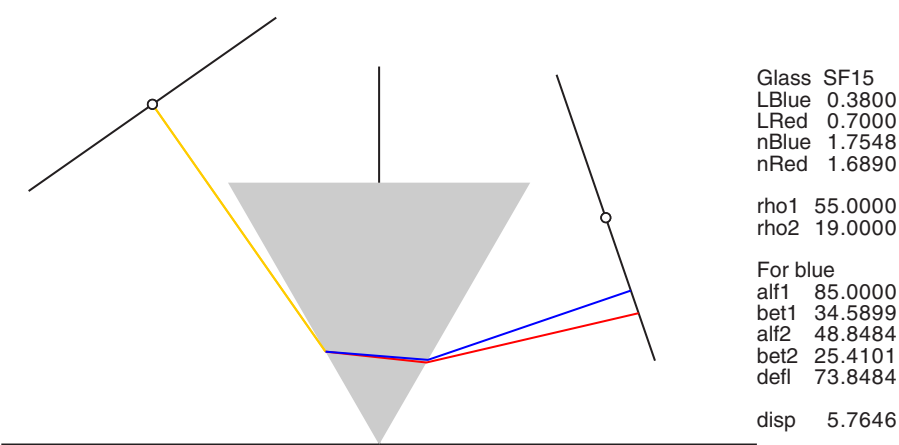
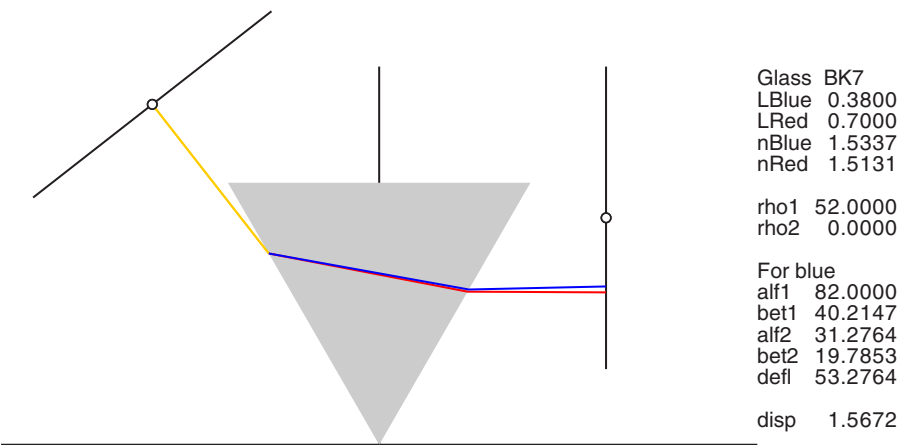
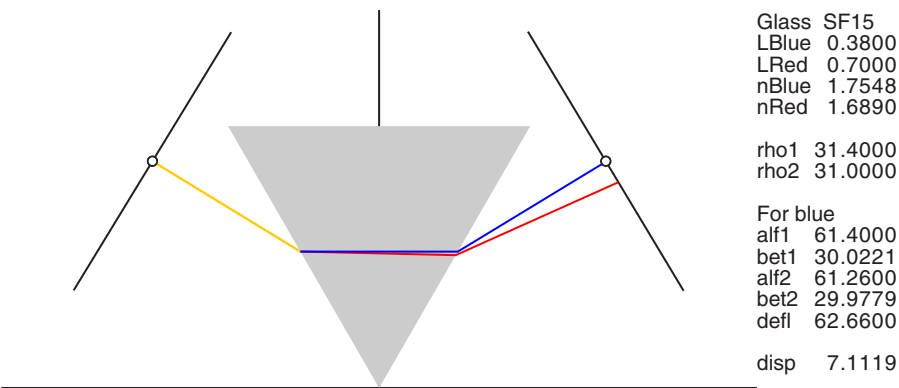
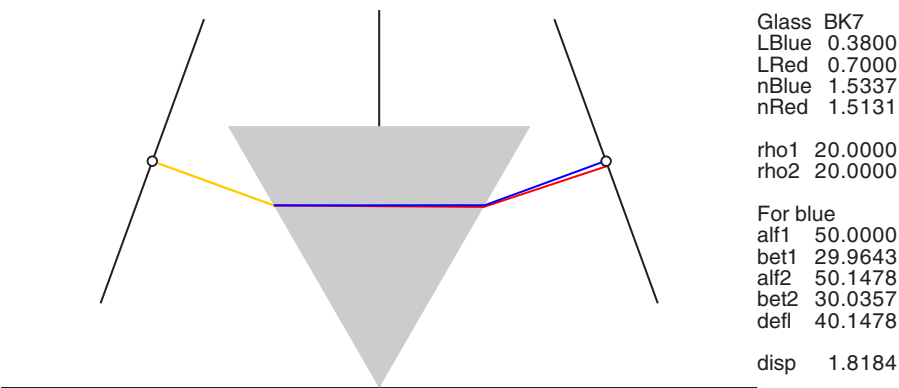
$$dA = d_{2x} u_{2y} - d_{2y} u_{2x}$$

$$dM = d_{2x} s_{2y} - d_{2y} s_{2x}$$

$$\mathbf{r}_2 = \mathbf{c}_2 + (dM / dA) \mathbf{u}_2$$

4. Geometry for maximal dispersion

For a practical prism experiment the dispersion should be as large as possible. Flint glass is the better choice. The optimal angles can be found by trial and error. The simulation below confirms the test results. Left crown glass, right flint glass. The upper two images show the symmetrical case. The lower show the optimal case. The output angle β_2 should be near to 90° for the shortest wavelength (violet). In the middle images one can see that large input angles α_1 do not deliver the best results.



Crown glass

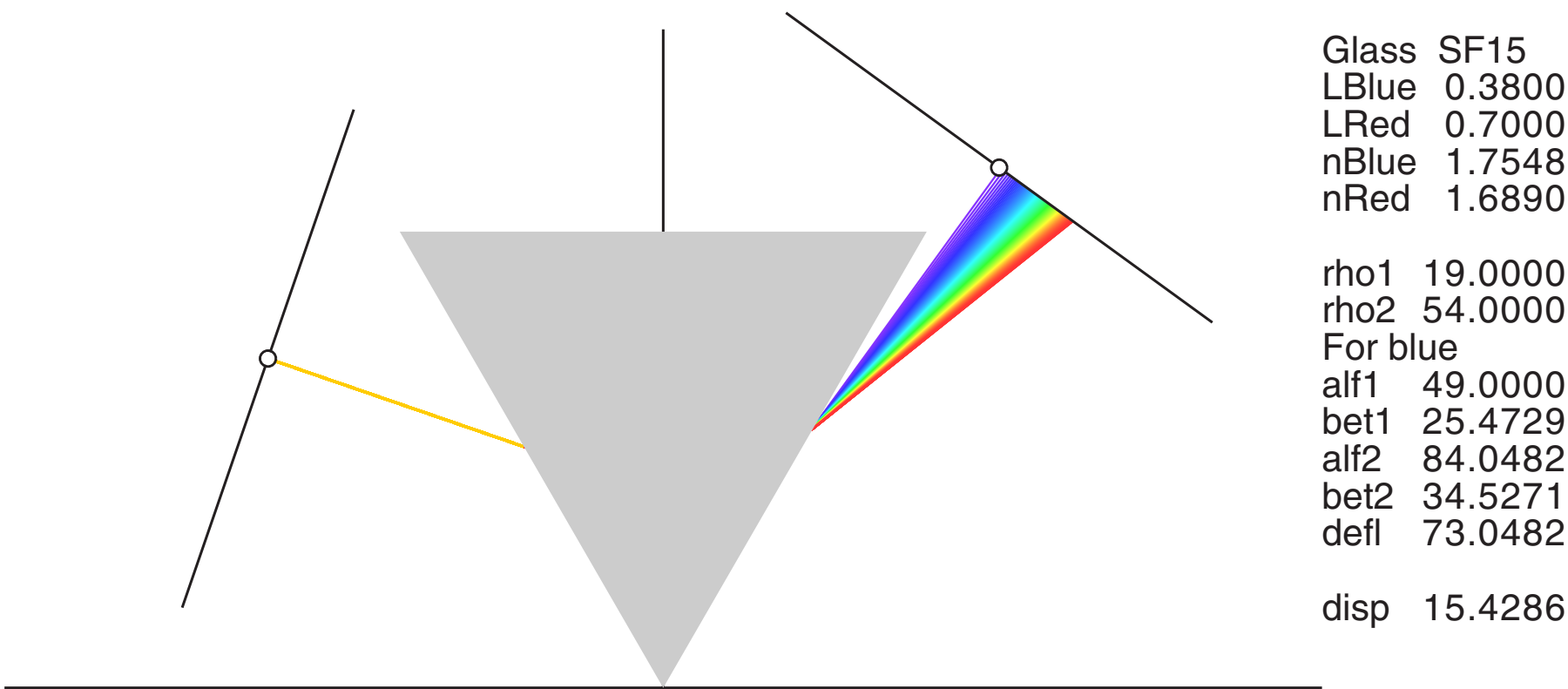
Flint glass

5. Ray visualization and a photo

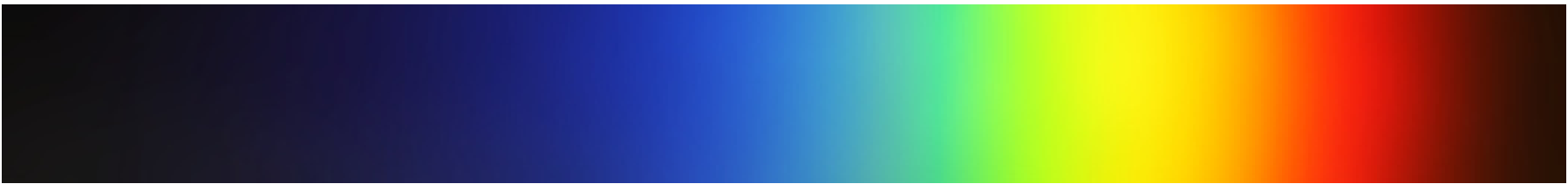
We are using throughout the document wavelengths from $0.380\mu\text{m}$ to $0.700\mu\text{m}$. This range reaches from almost invisible violet to almost invisible red.

The colors at the end of the spectrum can be made visible only by rather strong light sources in laboratories, $0.360\mu\text{m}$ to $0.800\mu\text{m}$.

A crude color visualization approximation is achieved by nonlinear HSB, a modification of the standard Hue-Saturation-Brightness model, which is represented by a cone [9], [24].



This is a photo of a real spectrum bar. A little image processing was applied.



6. Gamut limitation for sRGB

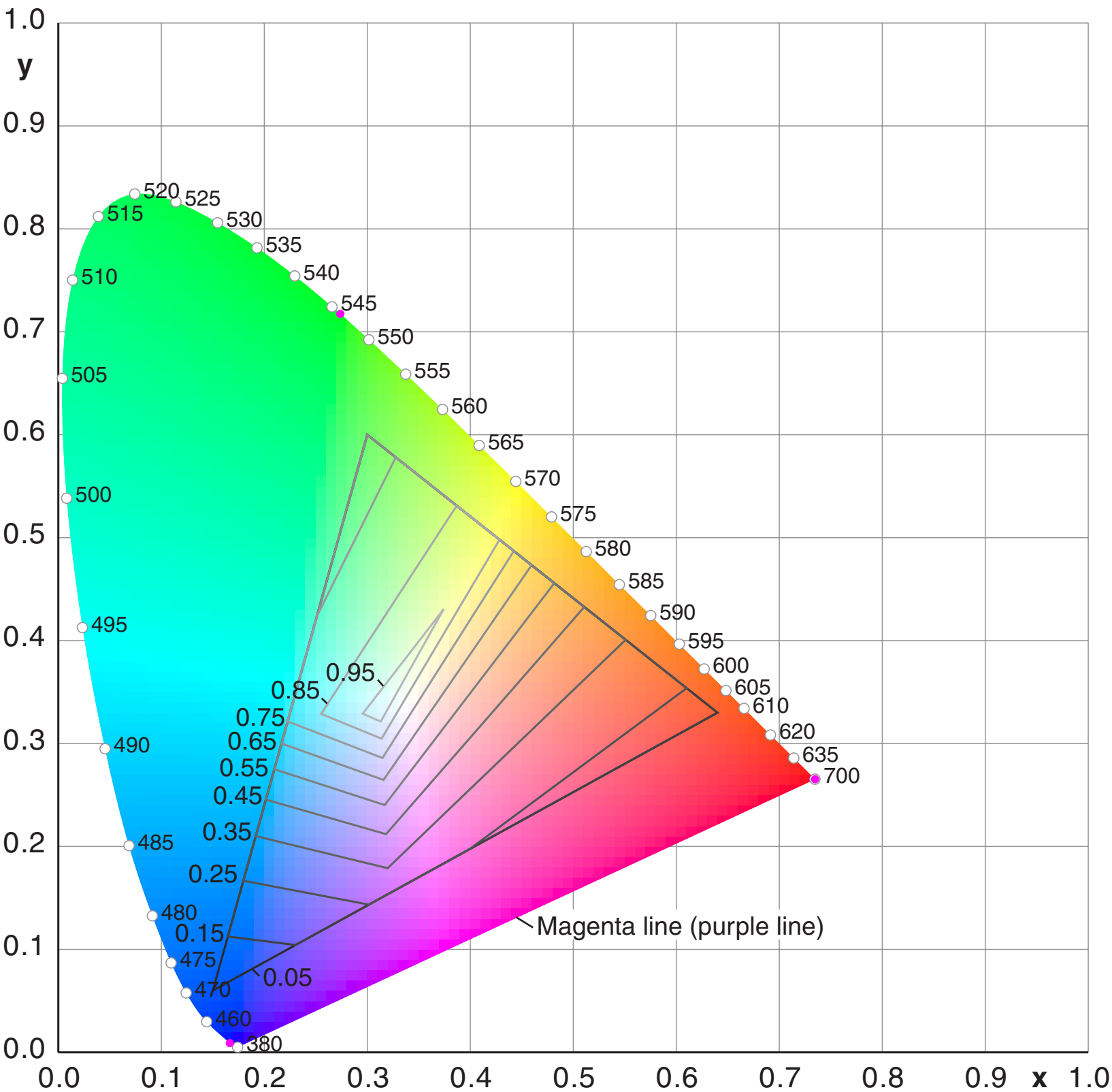
A correct visualization of the calculated spectrum bar would require media which can reproduce all spectral colors. This is not possible, not even by lasers. Especially one needs a plausible color reproduction on common cathode ray tube monitors (CRT monitors).

The chromaticity diagram CIE xyY below is a special perspective projection of the physical color space CIE XYZ onto a plane. The area, filled symbolically by colors, represents the human gamut, but the luminance is left out. Spectral colors are on the horseshoe contour.

The magenta line (purple line) shows mixtures of violet and red. Magenta itself is not a spectral color. The magenta line is practically a fiction because the end points are nearly invisible in real life. Practical magentas are mixed by blue and red.

Inside the horseshoe contour is the gamut triangle for sRGB (Rec.709 primaries and D65 white point), but this triangle is the projection of an affine distorted cube. The gamut depends on the luminance as well, which is shown for increasing luminances $Y = 0.05$ to 0.95 [22],[23].

sRGB is a standardized color space, a reasonable model for real CRT monitors. Mapping spectral colors to sRGB is obviously very difficult. Especially vibrant oranges are missing.



7. Gamut compression

7.1 Gamut compression Type A for single ray spectra in CIELab

If the spectrum is generated by one ray, then the visualization by sRGB requires optimally saturated colors and a certain visual balance as well.

If an orange cannot be shown brightly, then it is less convenient to show yellow in the neighbourhood as bright as possible.

The blue part looks always too bright. This well-known phenomenon will be explained later.

The gamut compression is done in CIELab [10],[21] by the author's simple method. More subtle strategies for gamut compressions are described e.g. by Ján Morovic in [14].

Find the hue in Lab by a,b (asterisks L*.. omitted)

Reduce the radius towards the L-axis until R,G,B are in-gamut

This delivers a_1, b_1, L_1

Modify L by

$$L_2 = 0.5 + 0.7(L - 0.5)$$

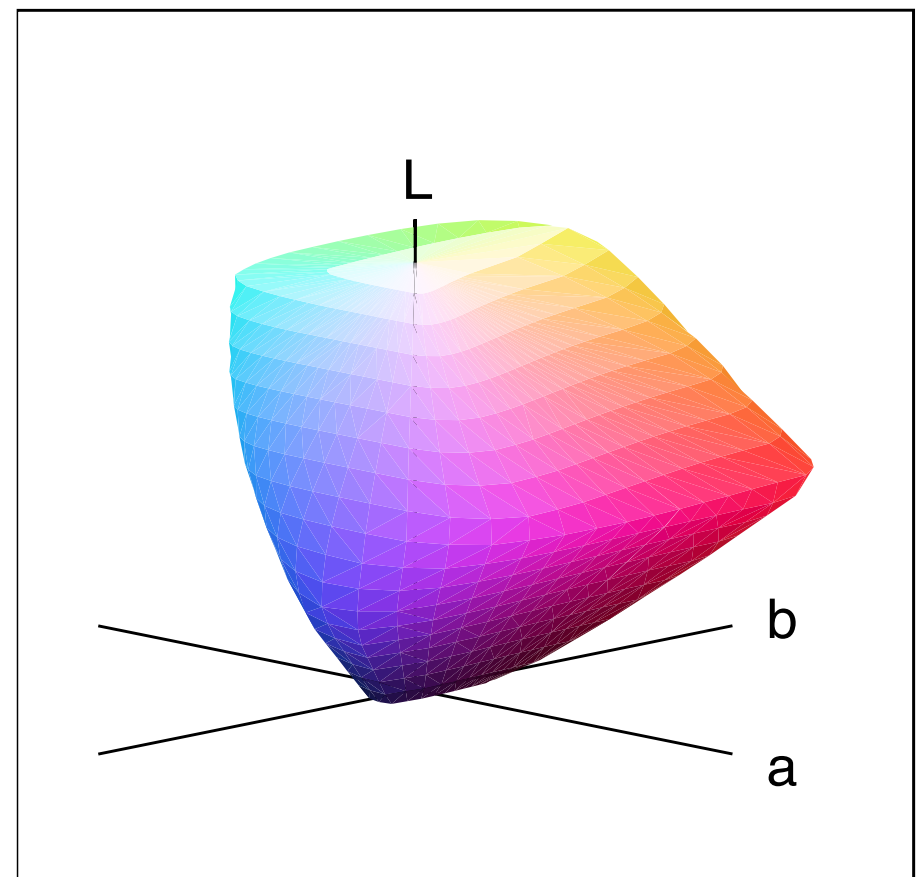
Reduce the radius towards the L-axis until R,G,B are in-gamut

This delivers a_2, b_2, L_2

Apply a linear interpolation between 1 and 2 and find the position with maximal saturation

$$s = \sqrt{a^2 + b^2} / L$$

The gamut compression will be improved later



7.2 Gamut compression Type B for multiple ray spectra in CIELab

The complete compression as above is not useful for spectra which are generated by several rays. In this case the light in the middle is more or less white and an increased saturation would be wrong.

The Lab luminance is simply multiplied by a factor of about 0.95 and only the first step of the compression is executed.

7.3 Gamut compression Type C by RGB clipping

Crude RGB clipping - even proportional clipping like here - does not deliver pleasant results.

If $R < 0$ then $R = 0$

If $G < 0$ then $G = 0$

If $B < 0$ then $B = 0$

$m = \max(R, G, B)$

If $m > 1$ Then

begin

$R = R/m$

$G = G/m$

$B = B/m$

end

8. Mapping the rays

The upper image shows again the crude color approximation by nonlinear HSB, now with a smaller number of color rays.

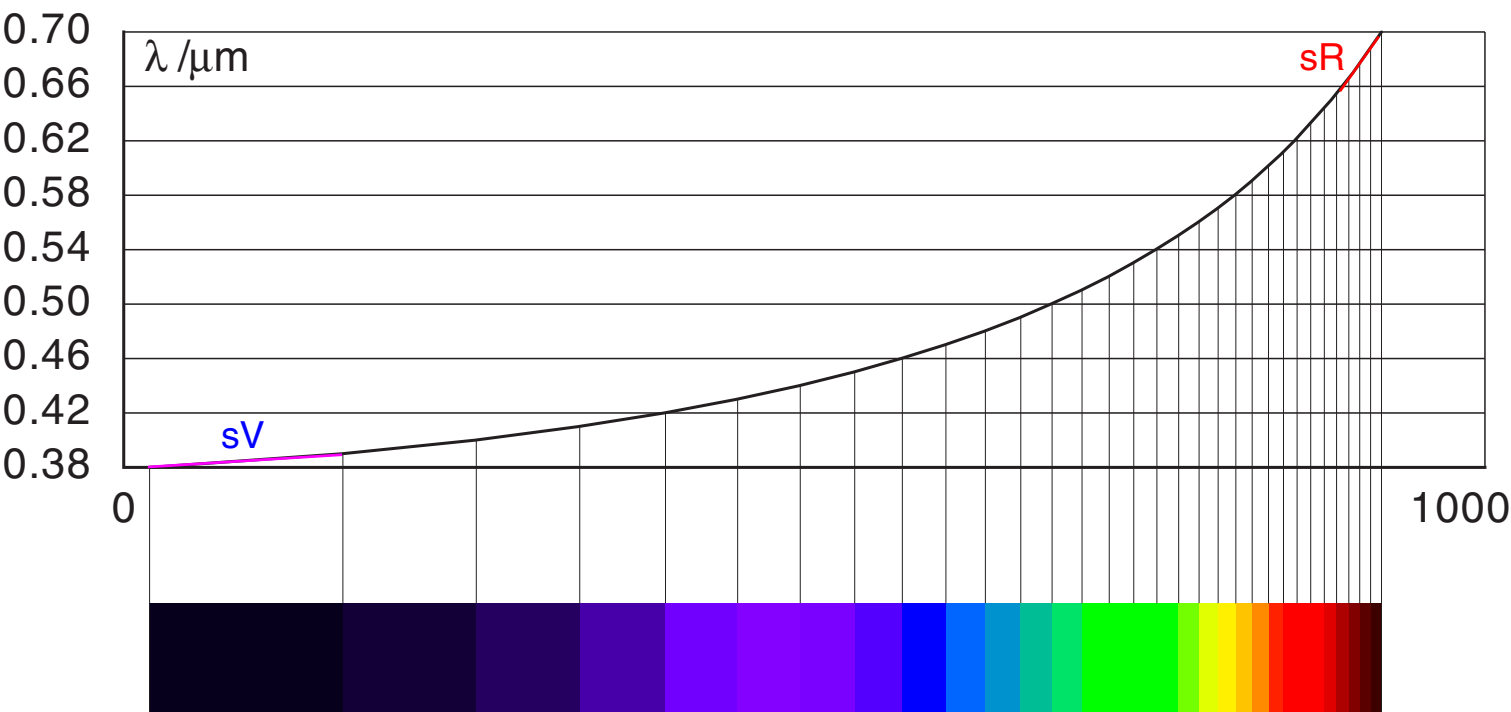
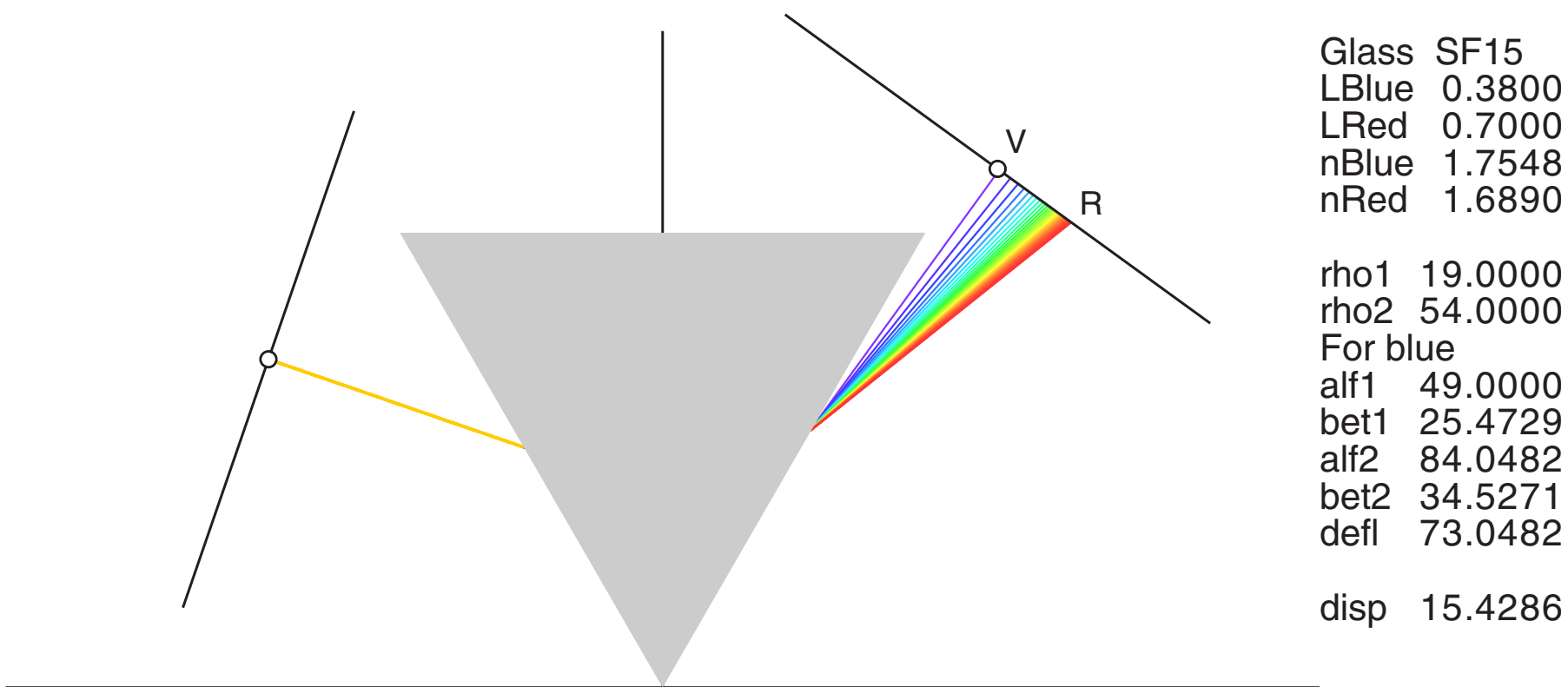
The lower image shows the content of an array with 1000+1 elements on the screen (right plane), starting approximately at the origin. Each color ray hits an element of this array.

The wavelength is written into the array. Gaps are filled by linear interpolation.

At position R we have many color rays per length, at position V fewer. In order to distribute the energy correctly across the length, the array content is multiplied by a slope factor. This is calculated by a linear interpolation of the slopes between sV and sR.

The content of the array is converted into the color space CIE XYZ, using the common color-matching functions, and then converted to sRGB [10],[21],[22].

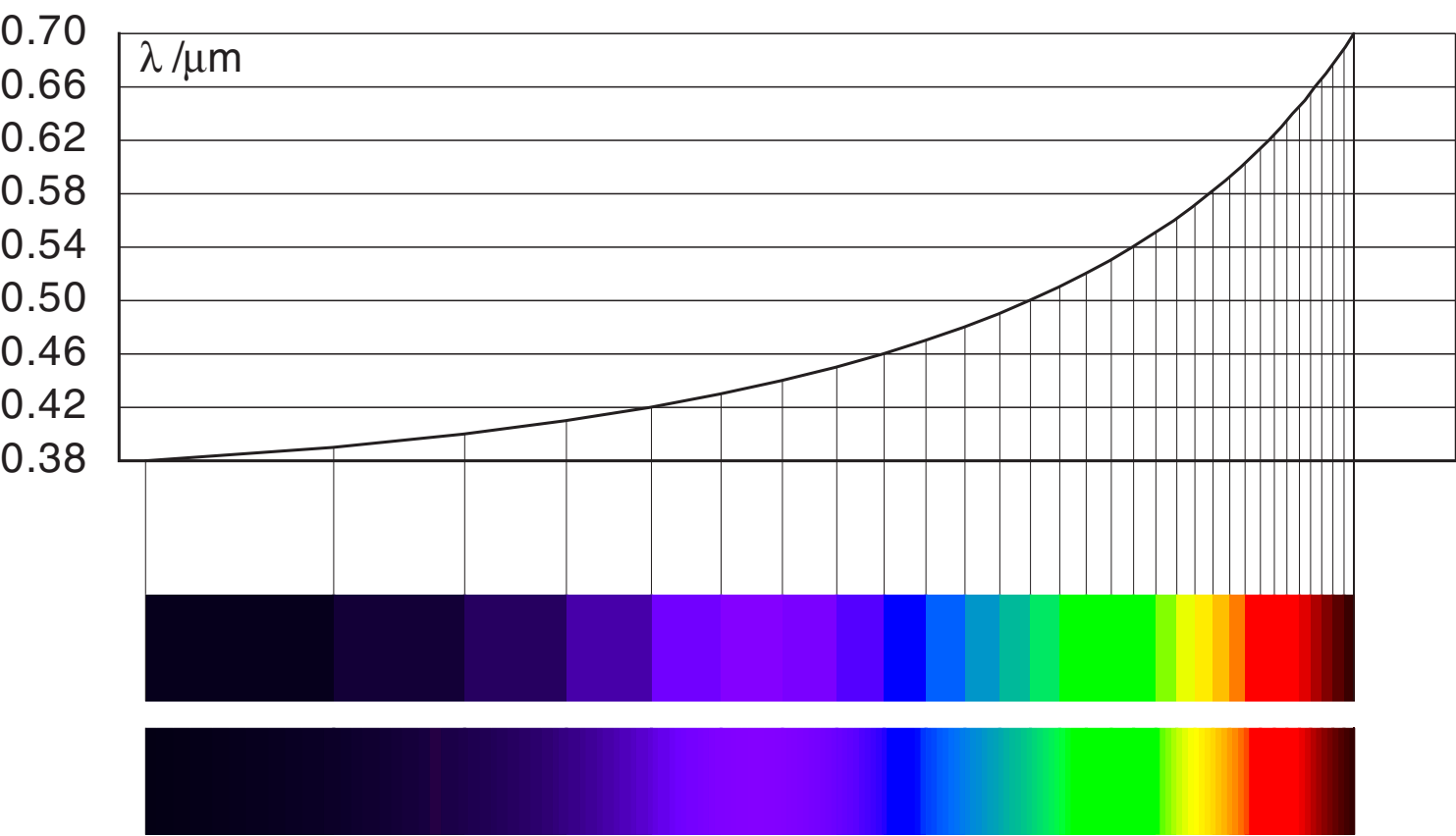
Compression is here done by Type C, RGB clipping. The spectrum bar is shown banded.



9. Single ray / Illuminant Equal Energy / Compression Type C / RGB clipping

Illuminant Equal Energy has a flat spectrum. Compression is done by Type C, RGB clipping. The spectrum bars are shown banded and continuous. The table shows linear RGB values, the bars were calculated with gamma encoding for sRGB. RGB clipping does not create balanced distributions of colors.

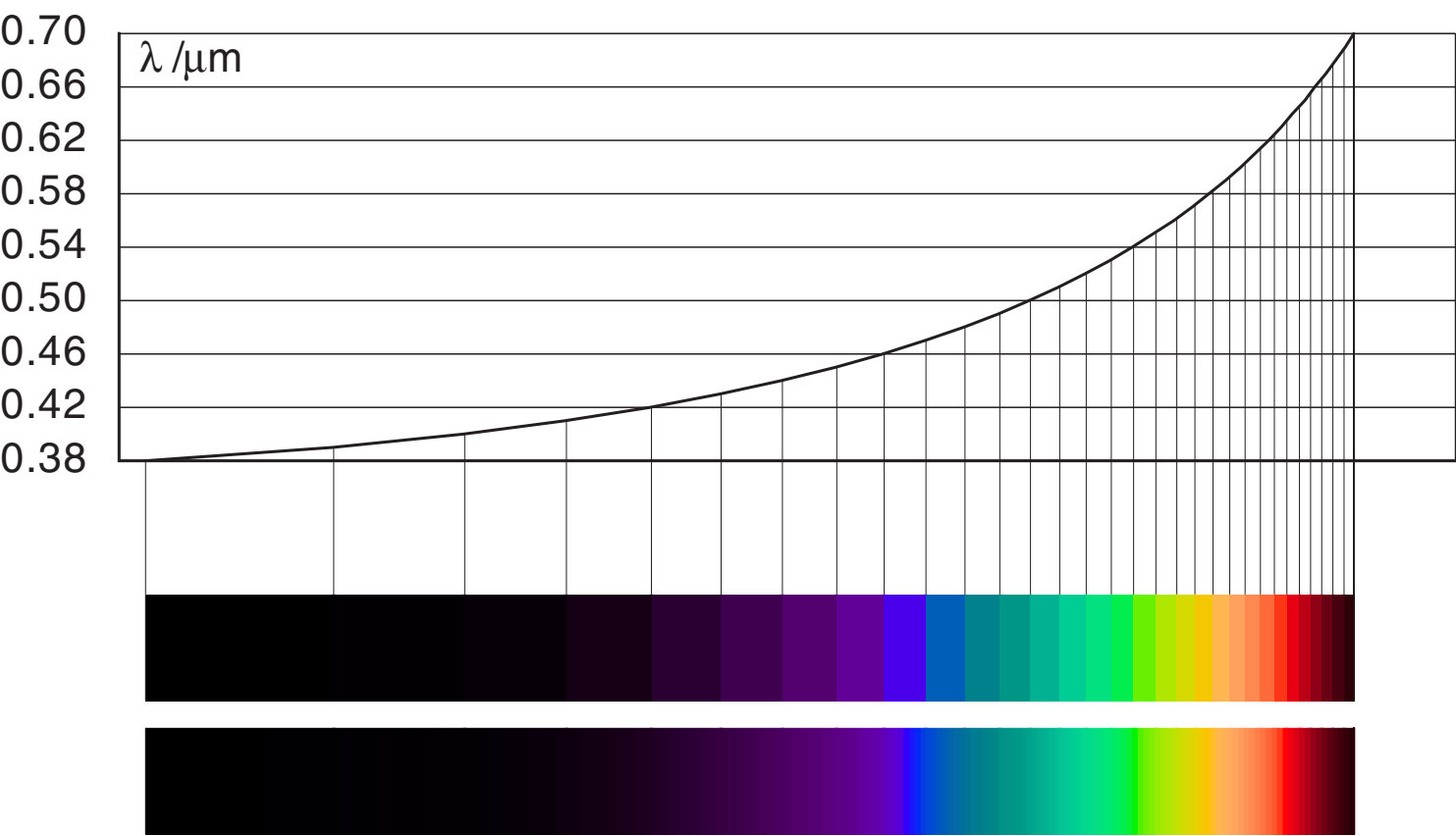
No RGB clipping, no RGB gamma encoding							With RGB clipping, no RGB gamma encoding						
Lam	L*	a*	b*	R	G	B	Lam	L*	a*	b*	R	G	B
0.38	0.0	5.5	-9.2	0.3	-0.3	1.8	0.38	0.7	4.2	-8.2	0.3	0.0	1.8
0.39	0.1	16.9	-25.0	0.9	-0.8	5.5	0.39	2.1	12.9	-22.0	0.9	0.0	5.5
0.40	0.4	53.0	-51.1	3.0	-2.6	18.5	0.40	7.0	34.1	-40.1	3.0	0.0	18.5
0.41	1.1	105.2	-85.6	9.1	-8.0	56.5	0.41	17.3	49.7	-58.4	9.1	0.0	56.5
0.42	3.6	175.9	-134.2	27.4	-24.5	175.7	0.42	32.4	72.4	-85.6	27.4	0.0	175.7
0.43	10.3	221.0	-171.4	53.9	-49.9	376.9	0.43	40.8	83.1	-93.5	53.9	0.0	255.0
0.44	17.0	215.6	-177.2	56.7	-56.6	474.6	0.44	41.1	83.3	-92.9	56.7	0.0	255.0
0.45	23.0	185.5	-168.0	37.6	-46.1	480.4	0.45	38.5	81.8	-97.4	37.6	0.0	255.0
0.46	29.4	141.2	-152.3	4.6	-25.5	450.9	0.46	33.1	79.5	-106.4	4.6	0.0	255.0
0.47	36.2	70.2	-121.5	-37.9	8.9	345.1	0.47	37.3	64.0	-99.5	0.0	8.9	255.0
0.48	44.1	-26.5	-77.8	-78.8	51.5	213.2	0.48	52.4	13.7	-64.0	0.0	51.5	213.2
0.49	52.7	-134.8	-32.1	-114.2	96.5	115.0	0.49	61.9	-30.4	-17.2	0.0	96.5	115.0
0.50	63.6	-254.0	11.3	-157.2	156.2	56.6	0.50	73.2	-60.1	26.0	0.0	156.2	56.6
0.51	76.3	-290.7	53.9	-209.6	240.0	16.6	0.51	85.9	-81.4	67.1	0.0	240.0	16.6
0.52	87.5	-243.4	95.3	-236.0	324.8	-14.9	0.52	87.7	-86.2	83.2	0.0	255.0	0.0
0.53	94.4	-196.6	122.7	-206.5	371.9	-31.1	0.53	87.7	-86.2	83.2	0.0	255.0	0.0
0.54	98.2	-155.4	143.9	-136.6	384.8	-40.0	0.54	87.7	-86.2	83.2	0.0	255.0	0.0
0.55	99.8	-114.3	159.6	-32.9	368.9	-43.2	0.55	87.7	-86.2	83.2	0.0	255.0	0.0
0.56	99.8	-71.6	166.5	100.8	329.1	-42.3	0.56	91.6	-54.9	87.9	100.8	255.0	0.0
0.57	98.1	-27.4	166.2	256.4	267.1	-38.1	0.57	97.1	-21.6	94.5	255.0	255.0	0.0
0.58	94.7	16.6	161.0	416.0	189.7	-31.8	0.58	89.1	-6.4	88.7	255.0	189.7	0.0
0.59	89.7	57.3	153.1	551.3	108.5	-24.5	0.59	77.1	19.0	80.4	255.0	108.5	0.0
0.60	83.5	90.0	142.8	630.4	39.3	-17.5	0.60	63.6	51.7	72.0	255.0	39.3	0.0
0.61	76.3	111.3	131.0	631.4	-7.2	-11.8	0.61	53.2	80.1	67.2	255.0	0.0	0.0
0.62	68.1	120.1	117.1	556.8	-28.9	-7.6	0.62	53.2	80.1	67.2	255.0	0.0	0.0
0.63	58.5	117.6	100.8	427.0	-32.0	-4.7	0.63	53.2	80.1	67.2	255.0	0.0	0.0
0.64	48.9	109.4	84.3	301.6	-27.0	-2.7	0.64	53.2	80.1	67.2	255.0	0.0	0.0
0.65	39.1	96.7	67.4	192.4	-18.9	-1.5	0.65	47.0	72.9	61.2	192.4	0.0	0.0
0.66	29.7	82.0	51.1	112.4	-11.6	-0.8	0.66	36.7	60.9	51.1	112.4	0.0	0.0
0.67	20.8	66.9	35.9	59.7	-6.3	-0.4	0.67	26.7	49.4	39.5	59.7	0.0	0.0
0.68	13.8	54.7	23.8	32.0	-3.4	-0.2	0.68	18.7	40.1	28.7	32.0	0.0	0.0
0.69	7.4	43.1	12.8	15.5	-1.7	-0.1	0.69	11.2	31.5	17.7	15.5	0.0	0.0
0.70	3.7	29.5	6.4	7.8	-0.9	-0.1	0.70	5.9	24.1	9.3	7.8	0.0	0.0



10. Single ray / Illuminant Equal Energy / Compr.Type A / CIELab

The same method as in the previous chapter, but now the compression is done by Type A in CIELab. The spectrum bars look better balanced. The table shows linear RGB values, the bars were calculated with gamma encoding for sRGB. The spectrum bars look better balanced because of CIELab gamut compression.

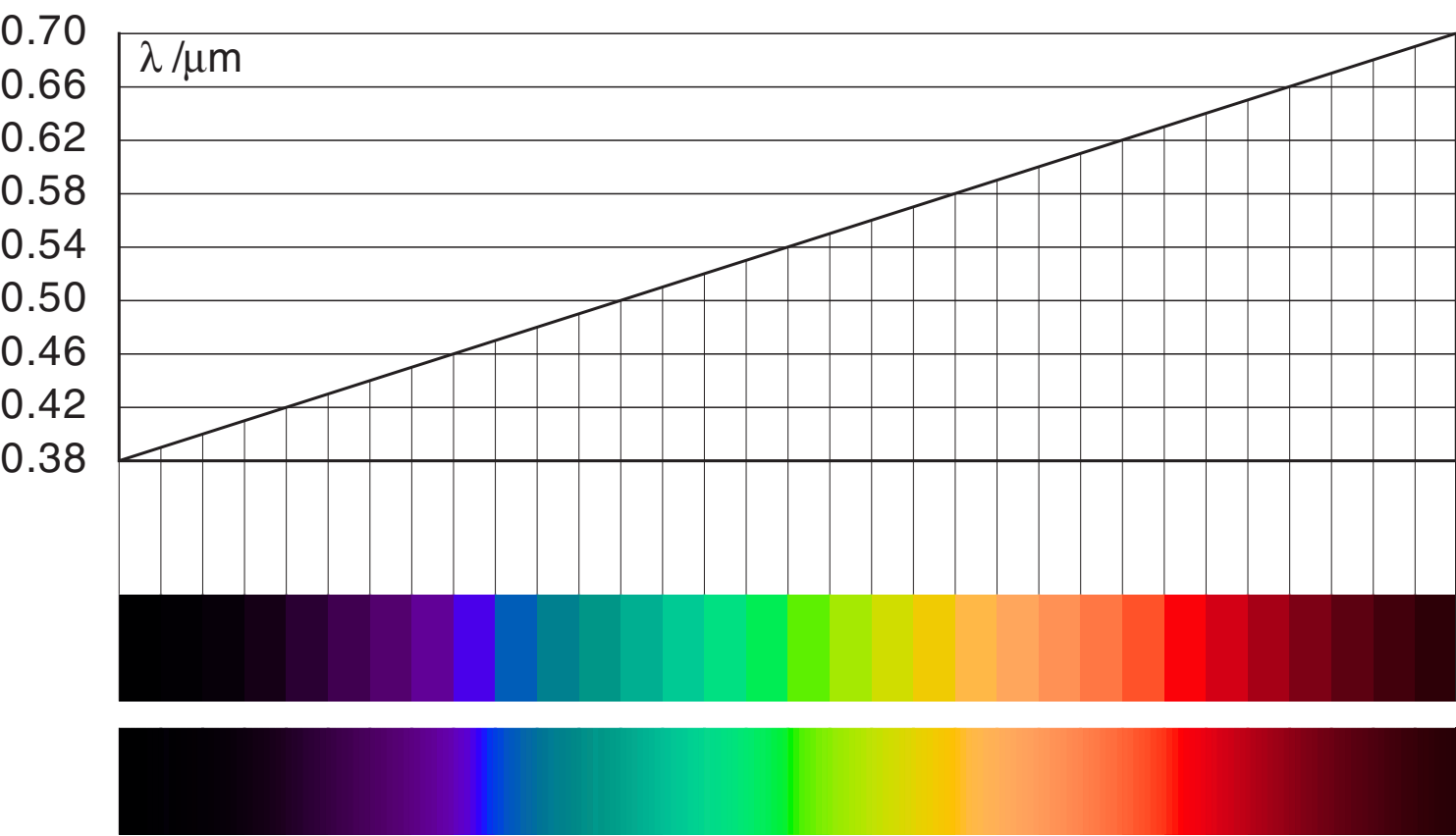
No gamut compression, no RGB gamma encoding							With gamut compression, no RGB gamma encoding						
Lam	L*	a*	b*	R	G	B	Lam	L*	a*	b*	R	G	B
0.38	0.0	5.5	-9.2	0.3	-0.3	1.8	0.38	0.0	0.2	-0.3	0.0	0.0	0.1
0.39	0.1	16.9	-25.0	0.9	-0.8	5.5	0.39	0.1	0.7	-1.0	0.1	0.0	0.2
0.40	0.4	53.0	-51.1	3.0	-2.6	18.5	0.40	0.4	2.0	-1.9	0.3	0.0	0.5
0.41	1.1	105.2	-85.6	9.1	-8.0	56.5	0.41	1.1	5.8	-4.7	1.1	0.0	1.2
0.42	3.6	175.9	-134.2	27.4	-24.5	175.7	0.42	3.6	18.9	-14.5	3.4	0.0	4.0
0.43	10.3	221.0	-171.4	53.9	-49.9	376.9	0.43	10.3	34.9	-27.1	9.3	0.0	13.6
0.44	17.0	215.6	-177.2	56.7	-56.6	474.6	0.44	17.0	44.4	-36.5	17.7	0.0	29.3
0.45	23.0	185.5	-168.0	37.6	-46.1	480.4	0.45	23.0	53.6	-48.5	26.7	0.0	56.1
0.46	29.4	141.2	-152.3	4.6	-25.5	450.9	0.46	29.4	65.6	-70.8	31.3	0.0	120.3
0.47	36.2	70.2	-121.5	-37.9	8.9	345.1	0.47	36.2	49.8	-86.3	0.0	12.5	198.2
0.48	44.1	-26.5	-77.8	-78.8	51.5	213.2	0.48	44.1	-10.4	-30.7	0.1	41.1	83.3
0.49	52.7	-134.8	-32.1	-114.2	96.5	115.0	0.49	51.9	-31.5	-7.5	0.0	65.3	62.5
0.50	63.6	-254.0	11.3	-157.2	156.2	56.6	0.50	59.5	-41.0	1.8	0.0	91.7	65.8
0.51	76.3	-290.7	53.9	-209.6	240.0	16.6	0.51	68.4	-49.5	9.2	0.1	129.5	76.2
0.52	87.5	-243.4	95.3	-236.0	324.8	-14.9	0.52	76.2	-60.7	23.8	0.2	172.3	68.8
0.53	94.4	-196.6	122.7	-206.5	371.9	-31.1	0.53	81.1	-71.9	44.9	0.0	204.8	41.5
0.54	98.2	-155.4	143.9	-136.6	384.8	-40.0	0.54	83.7	-82.3	76.1	0.0	226.4	3.7
0.55	99.8	-114.3	159.6	-32.9	368.9	-43.2	0.55	84.9	-58.9	82.2	62.6	215.8	0.0
0.56	99.8	-71.6	166.5	100.8	329.1	-42.3	0.56	84.9	-35.6	82.9	128.8	196.1	0.3
0.57	98.1	-27.4	166.2	256.4	267.1	-38.1	0.57	83.7	-13.8	83.6	191.4	169.3	0.0
0.58	94.7	16.6	161.0	416.0	189.7	-31.8	0.58	81.3	8.5	82.8	250.6	136.0	0.3
0.59	89.7	57.3	153.1	551.3	108.5	-24.5	0.59	77.8	20.9	56.0	255.0	110.1	24.2
0.60	83.5	90.0	142.8	630.4	39.3	-17.5	0.60	73.4	31.0	49.2	254.5	85.1	26.4
0.61	76.3	111.3	131.0	631.4	-7.2	-11.8	0.61	68.4	42.2	49.7	254.3	59.7	19.4
0.62	68.1	120.1	117.1	556.8	-28.9	-7.6	0.62	62.7	55.6	54.3	254.9	34.4	10.2
0.63	58.5	117.6	100.8	427.0	-32.0	-4.7	0.63	56.0	72.7	62.3	254.9	9.1	2.3
0.64	48.9	109.4	84.3	301.6	-27.0	-2.7	0.64	49.2	75.8	58.4	213.4	0.0	1.5
0.65	39.1	96.7	67.4	192.4	-18.9	-1.5	0.65	39.1	63.8	44.4	127.5	0.0	2.1
0.66	29.7	82.0	51.1	112.4	-11.6	-0.8	0.66	29.7	53.4	33.3	72.9	0.0	2.0
0.67	20.8	66.9	35.9	59.7	-6.3	-0.4	0.67	20.8	43.0	23.1	37.9	0.0	1.6
0.68	13.8	54.7	23.8	32.0	-3.4	-0.2	0.68	13.8	34.7	15.1	19.9	0.0	1.2
0.69	7.4	43.1	12.8	15.5	-1.7	-0.1	0.69	7.4	27.3	8.1	9.6	0.0	0.7
0.70	3.7	29.5	6.4	7.8	-0.9	-0.1	0.70	3.7	17.2	3.7	4.8	0.0	0.4



11. Single Ray / Linear Spectrum / Illuminant Equal Energy / Compr. Type A / CIELab

This is *not* the prism experiment. The wavelength is now linearly distributed along the horizontal axis. Graphics like this appear often in text books, mostly wrong though. Modern spectrophotometers use gratings instead of prisms. Then the bar would look like here. The table shows linear RGB values. Instead of Lab values (the same as on the previous page) the CIE XYZ values are shown. The bars were calculated with gamma encoding for sRGB.

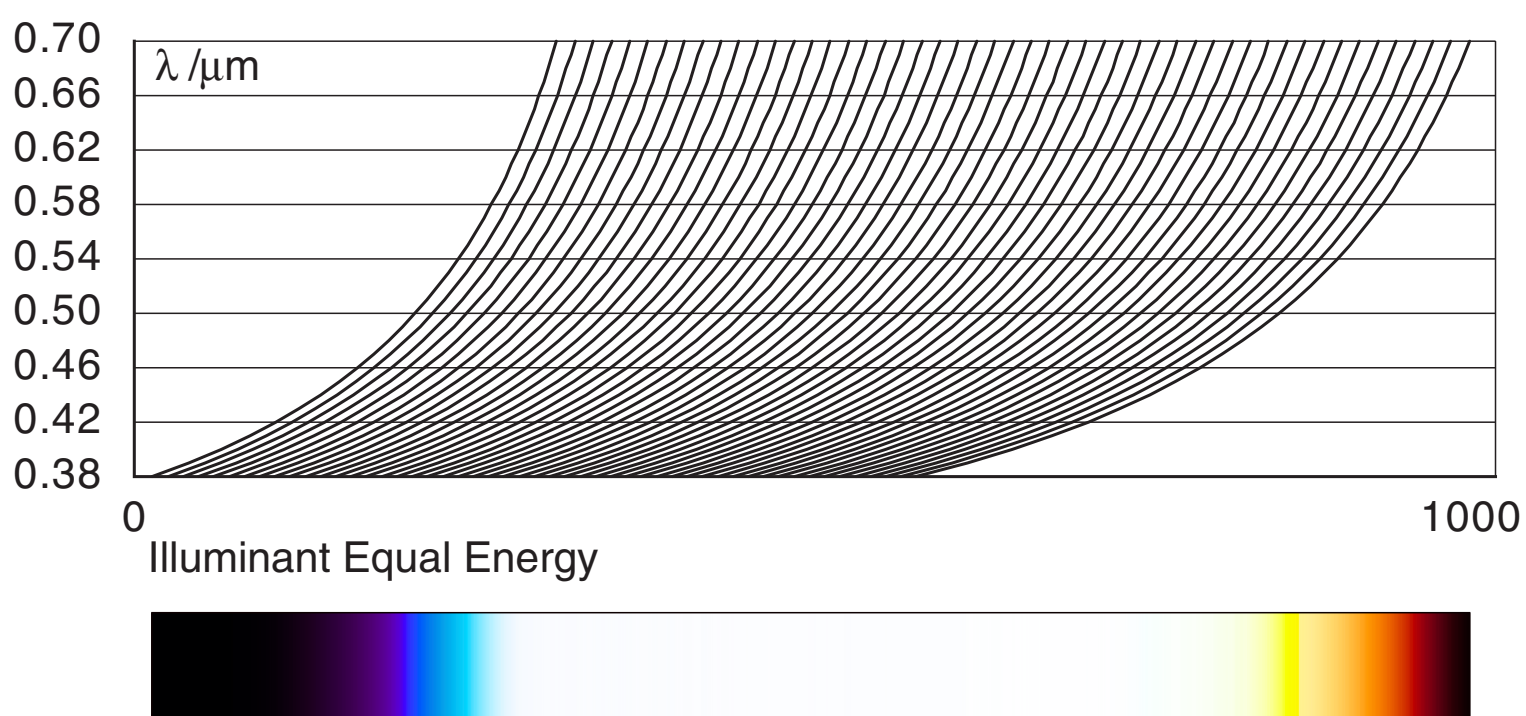
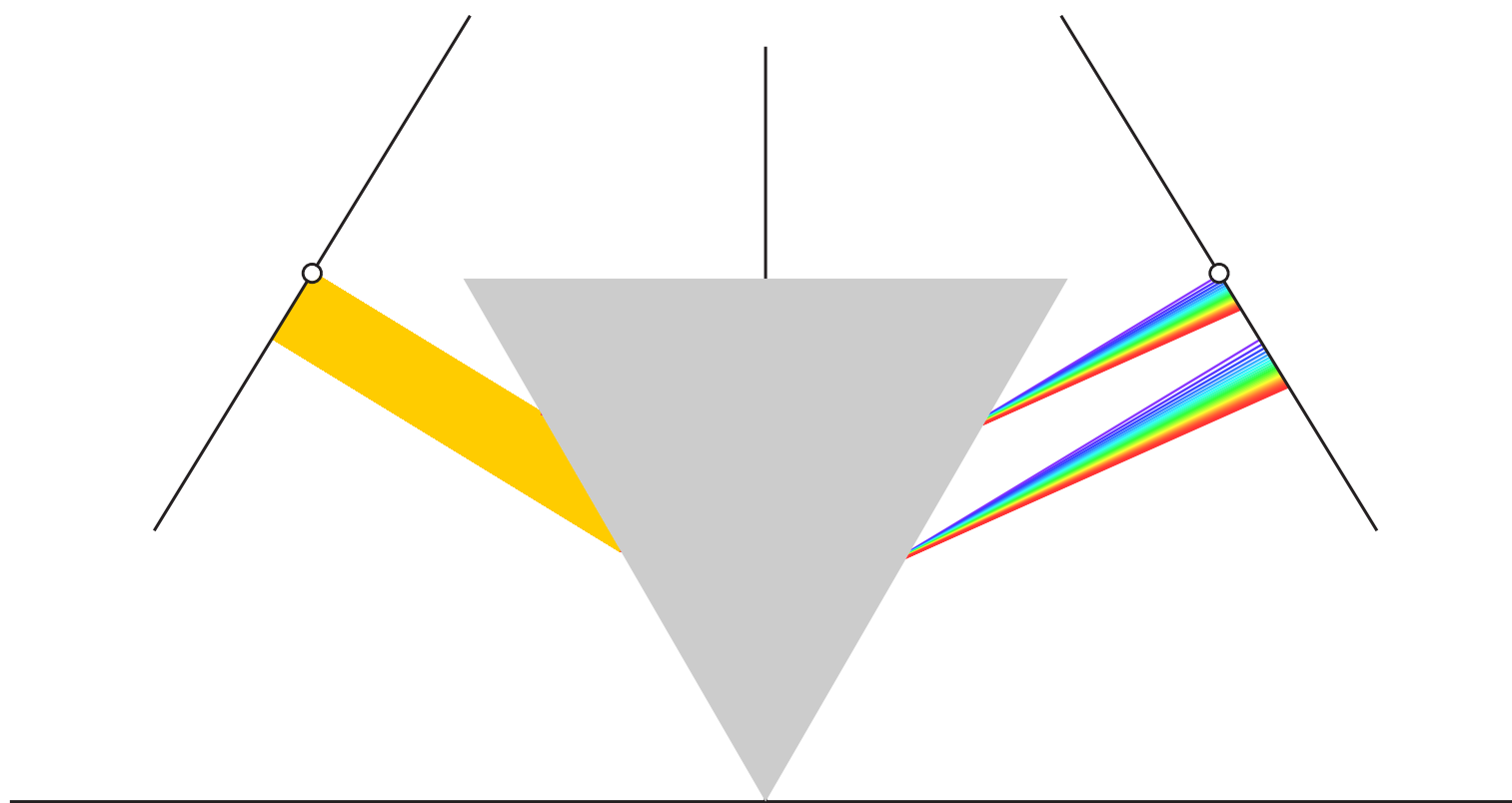
No gamut compression, no RGB gamma encoding							With gamut compression, no RGB gamma encoding						
Lam	X•100	Y•100	Z•100	R	G	B	Lam	X•100	Y•100	Z•100	R	G	B
0.38	0.1	0.0	0.6	0.3	-0.3	1.8	0.38	0.0	0.0	0.0	0.0	0.0	0.1
0.39	0.4	0.0	2.0	0.9	-0.8	5.5	0.39	0.0	0.0	0.1	0.1	0.0	0.2
0.40	1.4	0.0	6.8	3.0	-2.6	18.5	0.40	0.1	0.0	0.2	0.3	0.0	0.5
0.41	4.4	0.1	20.7	9.1	-8.0	56.5	0.41	0.3	0.1	0.5	1.1	0.0	1.2
0.42	13.4	0.4	64.6	27.4	-24.5	175.7	0.42	0.8	0.4	1.5	3.4	0.0	4.0
0.43	28.4	1.2	138.6	53.9	-49.9	376.9	0.43	2.5	1.2	5.2	9.3	0.0	13.6
0.44	34.8	2.3	174.7	56.7	-56.6	474.6	0.44	4.9	2.3	11.1	17.7	0.0	29.3
0.45	33.6	3.8	177.2	37.6	-46.1	480.4	0.45	8.3	3.8	21.1	26.7	0.0	56.1
0.46	29.1	6.0	166.9	4.6	-25.5	450.9	0.46	13.6	6.0	45.1	31.3	0.0	120.3
0.47	19.5	9.1	128.8	-37.9	8.9	345.1	0.47	15.8	9.1	74.5	0.0	12.5	198.2
0.48	9.6	13.9	81.3	-78.8	51.5	213.2	0.48	11.7	13.9	33.0	0.1	41.1	83.3
0.49	3.2	20.8	46.5	-114.2	96.5	115.0	0.49	13.6	20.1	26.3	0.0	65.3	62.5
0.50	0.5	32.3	27.2	-157.2	156.2	56.6	0.50	17.5	27.6	28.8	0.0	91.7	65.8
0.51	0.9	50.3	15.8	-209.6	240.0	16.6	0.51	23.6	38.5	34.5	0.1	129.5	76.2
0.52	6.3	71.0	7.8	-236.0	324.8	-14.9	0.52	29.1	50.3	33.7	0.2	172.3	68.8
0.53	16.5	86.2	4.2	-206.5	371.9	-31.1	0.53	31.7	58.6	25.0	0.0	204.8	41.5
0.54	29.0	95.4	2.0	-136.6	384.8	-40.0	0.54	32.0	63.6	12.0	0.0	226.4	3.7
0.55	43.3	99.5	0.9	-32.9	368.9	-43.2	0.55	40.4	65.7	10.6	62.6	215.8	0.0
0.56	59.4	99.5	0.4	100.8	329.1	-42.3	0.56	48.3	65.7	10.3	128.8	196.1	0.3
0.57	76.2	95.2	0.2	256.4	267.1	-38.1	0.57	54.7	63.5	9.4	191.4	169.3	0.0
0.58	91.6	87.0	0.2	416.0	189.7	-31.8	0.58	59.6	59.0	8.4	250.6	136.0	0.3
0.59	102.6	75.7	0.1	551.3	108.5	-24.5	0.59	58.4	52.8	16.1	255.0	110.1	24.2
0.60	106.2	63.1	0.1	630.4	39.3	-17.5	0.60	55.0	45.8	15.7	254.5	85.1	26.4
0.61	100.3	50.3	0.0	631.4	-7.2	-11.8	0.61	50.9	38.5	12.0	254.3	59.7	19.4
0.62	85.4	38.1	0.0	556.8	-28.9	-7.6	0.62	46.8	31.2	7.3	254.9	34.4	10.2
0.63	64.2	26.5	0.0	427.0	-32.0	-4.7	0.63	42.7	23.9	3.2	254.9	9.1	2.3
0.64	44.8	17.5	0.0	301.6	-27.0	-2.7	0.64	34.6	17.8	2.2	213.4	0.0	1.5
0.65	28.3	10.7	0.0	192.4	-18.9	-1.5	0.65	20.8	10.7	1.8	127.5	0.0	2.1
0.66	16.5	6.1	0.0	112.4	-11.6	-0.8	0.66	11.9	6.1	1.3	72.9	0.0	2.0
0.67	8.7	3.2	-0.0	59.7	-6.3	-0.4	0.67	6.3	3.2	0.9	37.9	0.0	1.6
0.68	4.7	1.7	0.0	32.0	-3.4	-0.2	0.68	3.3	1.7	0.6	19.9	0.0	1.2
0.69	2.3	0.8	-0.0	15.5	-1.7	-0.1	0.69	1.6	0.8	0.3	9.6	0.0	0.7
0.70	1.1	0.4	-0.0	7.8	-0.9	-0.1	0.70	0.8	0.4	0.2	4.8	0.0	0.4



12. Multiple ray / Illuminant Equal Energy / Compr.Type B / CIELab

A continuous light band with flat spectrum is simulated by multiple rays. Each of them is interpreted according to chapter 8. The content of the interpolated measurement array is multiplied by the color-matching functions and the slope function. Then summed up in a second array which contains CIE XYZ values for each wavelength. The content of the XYZ array is normalized for $Y_{\max}=1$ after finishing the measurement. The sum approximates an integration.

In the middle we have white light. At the ends we can see color fringes. This is Goethe's important observation on which he based his attack against Newton: colors appear at the boundaries of light and shadow.



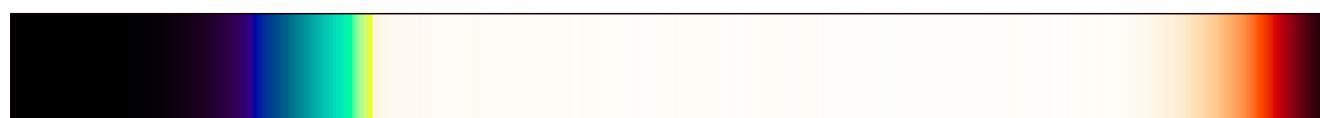
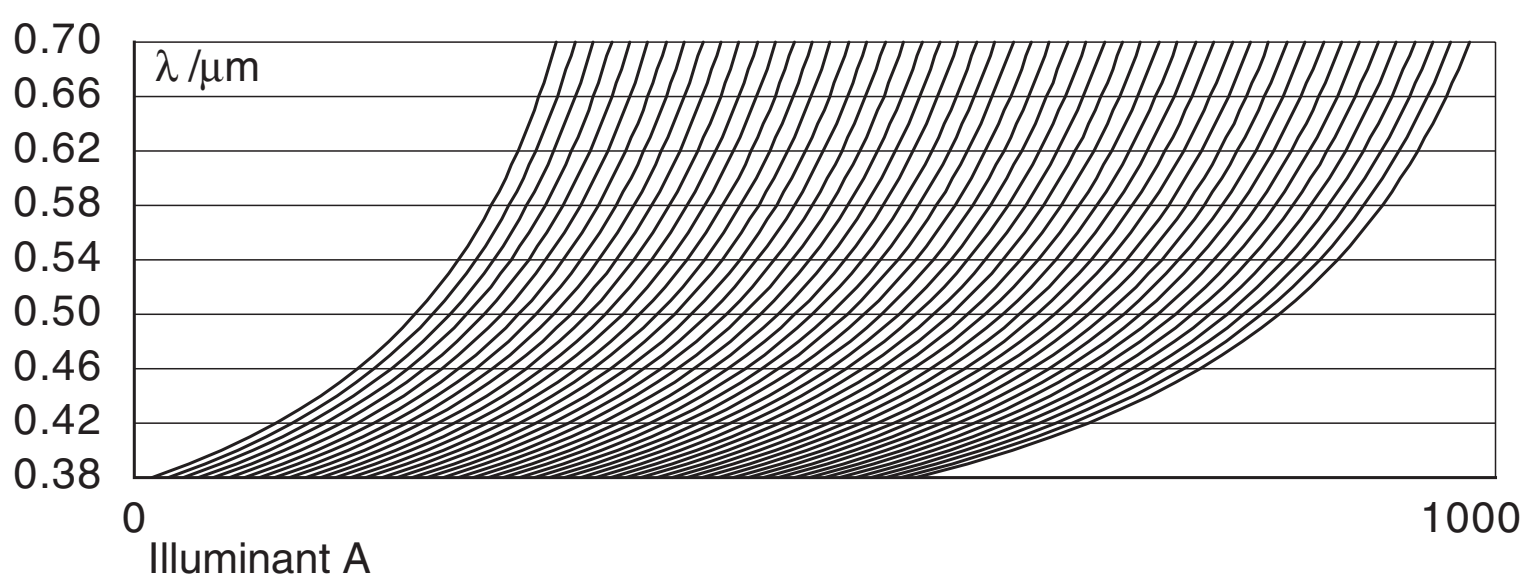
13. Multiple ray / Illuminants A, D50, D65, D75 / Compr. Type B / CIE Lab

Different illuminants generate different shades of white in the middle. Illuminant A is a Planckian radiator with color temperature 2856 K, as produced by a domestic tungsten bulb. Candle light would look even ‘warmer’, which means it has a lower color temperature, about 1900 K. Graphics for typical spectra are found in books [10] and in documents by the author [19].

The values in the measurement array are multiplied by one of these spectra as well, in addition to the multiplication by color-matching functions and slope functions.

In real life the discrepancies for different illuminants would be less obvious because of adaptation: the tungsten or candle yellow would look less yellow-ish and north skylight D75 would look less blue-ish.

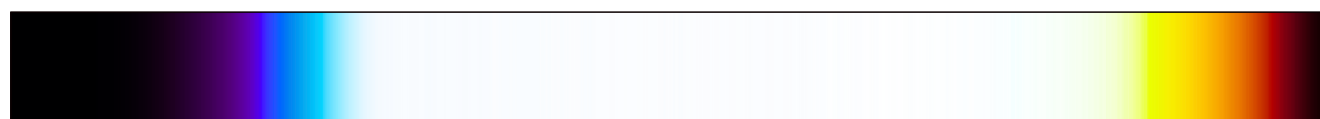
The simulation of perceived colors is handled in color theory by chromatic adaptation transforms [10],[14],[21]. These are *not* applied here.



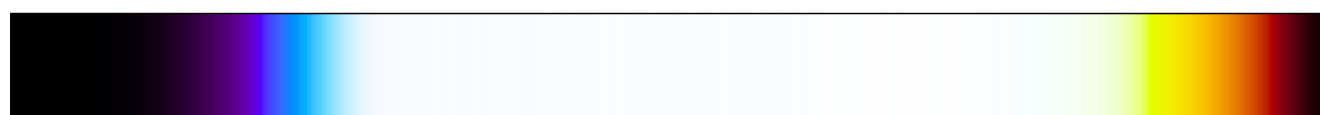
Illuminant D50



Illuminant D65



Illuminant D75



14. Goethe's explanations

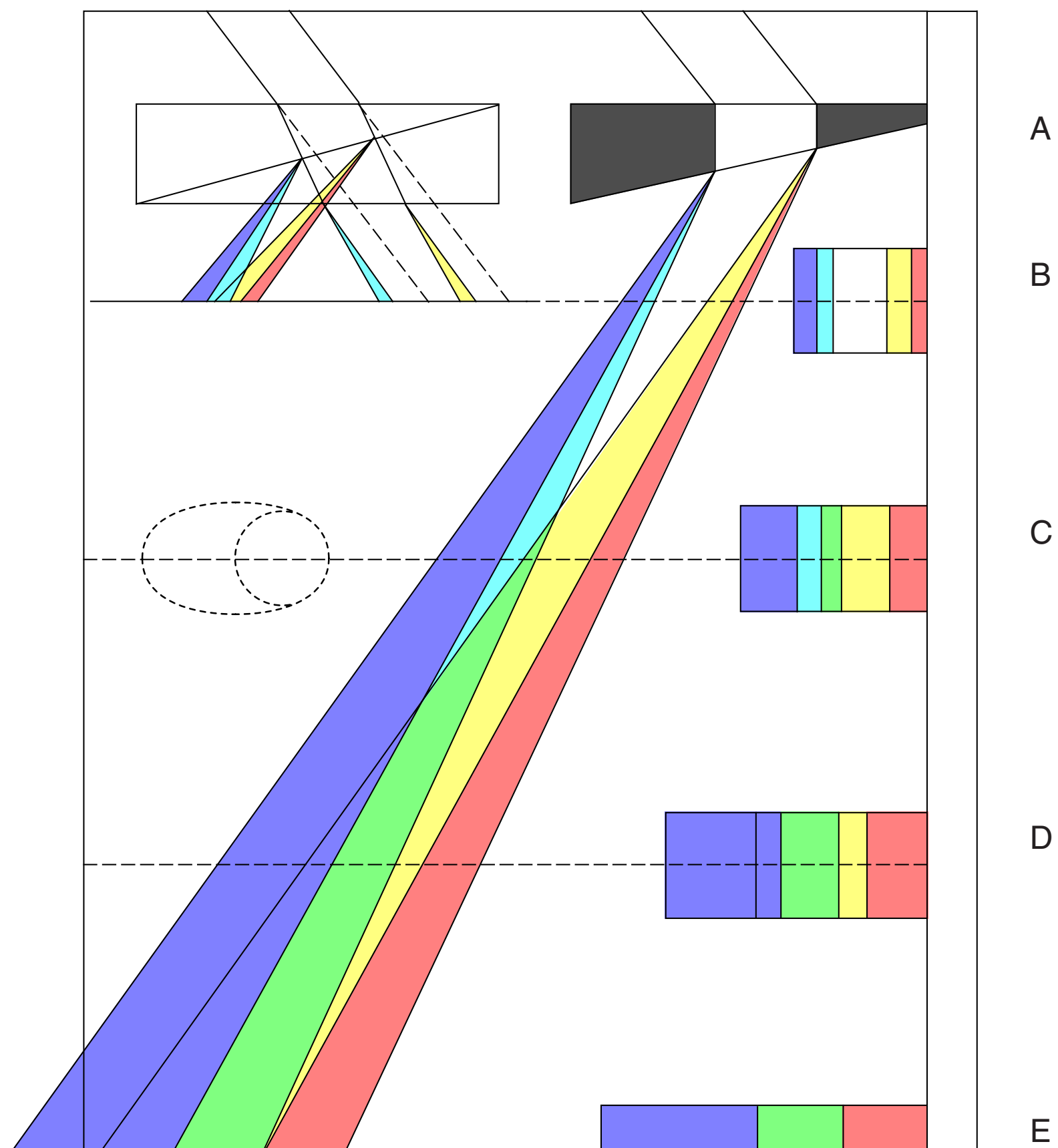
The graphic shows Goethe's *Fünfte Tafel*. Here it is a new drawing by the author, based on a print in [2], as accurate as possible.

The small image top left visualizes three situations for a light band, represented by two rays: without refraction, refracted by a parallel plate and refracted by the upper prism. Color fringes are emphasized. The large image should make the refraction effect by one prism much clearer. The ray geometry in the prism (A) is wrong, but this is more a symbolical drawing which shows 'light versus shadow'.

The cross-section (B) is approximately the same as in the previous chapters. But then said Goethe: 'the cross-section, where the ellipse is drawn, is about the one where Newton and his followers perceive, fix and measure the image'.

Does it mean that Goethe expected a spectrum without white in the middle, like in the cross-sections (C) to (E) ? Just an overlay of edge effects ? It seems so, but then he were terribly wrong - the rays in the middle cannot be ignored.

Goethe used a light band, Newton used for his *crucial experiments* a thin ray. And he knew already that light bands deliver white in the middle.



15. Conclusions

The Conclusions refer to the Introduction, chapter 1. Further on some remarks on the perceived brightness of blue are added.

A [2, *Zweiter Teil, Konfession des Verfassers*]

Aber wie verwundert war ich, ...'

A prism refracts a light band or a dense bundle of (fictitious) rays without strong color effects in the middle because these rays sum up to white light. Goethe was not aware of this superposition. Therefore he emphasized color effects correctly at the boundary of the bundle. Newton's *crucial experiments* were essentially done with thin rays.

It is not understandable why Goethe did not discuss the case for a thin ray. It seems that he regarded this as a quite unnatural test condition which had nothing to do with real life.

In fact he did not read *Opticks* carefully or at all. For example, the ellipse (C) in his drawing was clearly described by Newton as 'oblong', the envelop of a sequence of circles.

B [2, *Zweiter Teil, Polemischer Teil*]

'Also, um beim Refraktionsfalle zu verweilen, ...'

Goethe assumes that refraction itself does not create color effects. This would require an image which is refracted and shifted as well, and which has boundaries, because it is an image. But if it is refracted then it is shifted - the argument is not understandable.

On the other hand he is right in a certain sense, though without being aware of: refraction itself does not colorize anything. The color or dispersion is a result of different refraction indices for different wavelengths, for different spectral colors.

C [2, *Zweiter Teil, Beiträge zur Optik*]

'Unter den eigentlichen farbigen Erscheinungen sind nur zwei, ...'

Goethe's *Erste Tafel* shows color wheels. It seems that he had accepted Newton's model to a certain extent.

Goethe's model uses (also according to [7]) three primaries yellow, blue, red and three secondaries orange, violet, green. On the other hand he considers only yellow and blue as pure colors. Maybe this was just an unnecessary complication.

Goethe knew almost all the other contributors to color science, as mentioned in the same book [7], but not yet *Philipp Otto Runge*, who published 1810 the first three-dimensional color space, a sphere.

The Blue Mystery

In simulations we perceive in the spectrum bars the small part for pure blue on monitors as too bright. The color reproduction is based on the sRGB model which describes with sufficient accuracy common cathode ray tube monitors (real monitors are calibrated by instruments, a process which introduces the actual parameters into the color management system).

The contribution of blue to the luminance is given by equations like this:

$$L = 0.3R + 0.6G + 0.1B$$

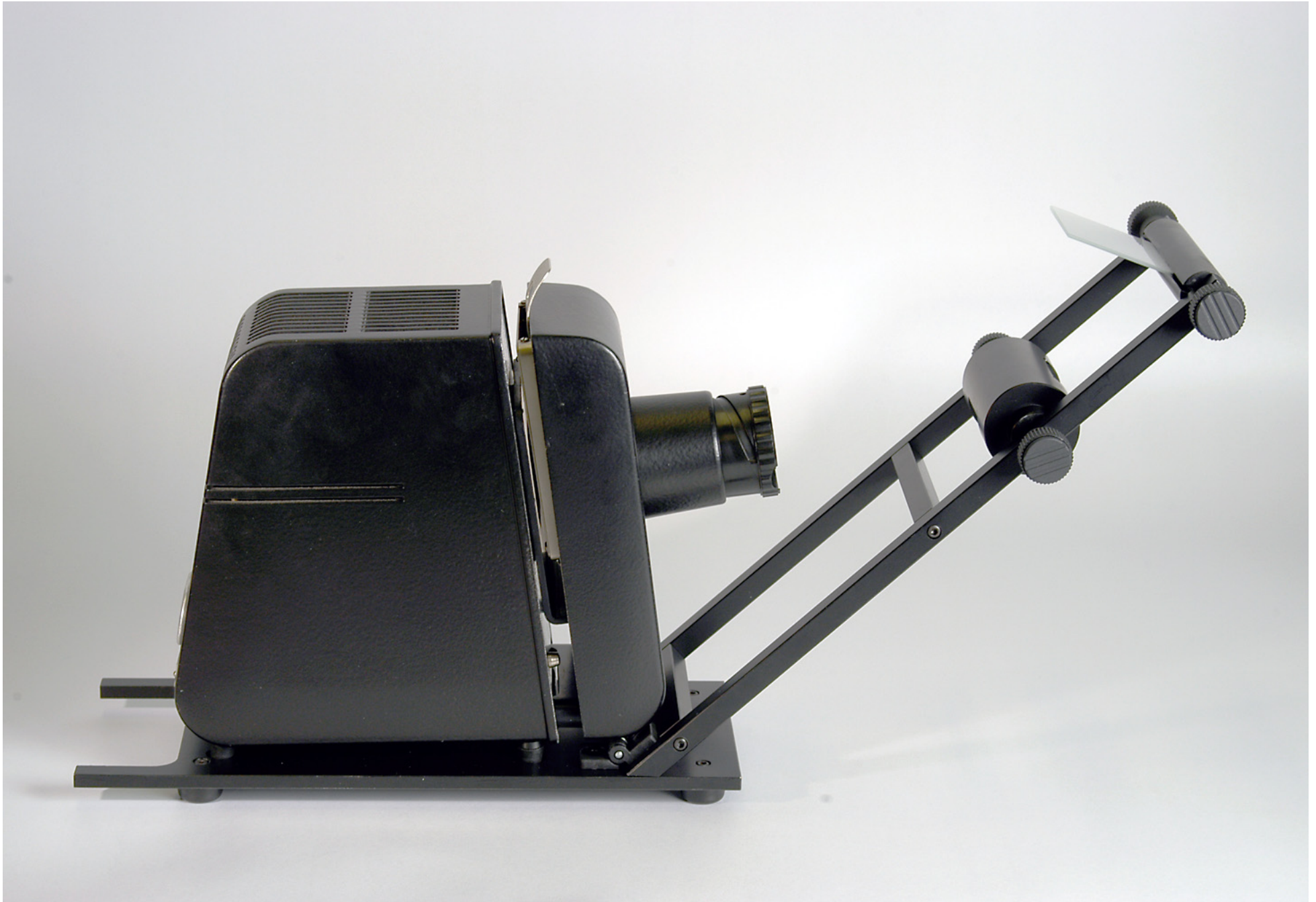
With accurate numbers this is colorimetrically correct. Is blue really six or seven times darker than green? The author could not prove such a ratio by flicker tests.

An explanation might be the *Helmholtz-Kohlrausch effect* [13] which means: fully saturated colors appear brighter than less saturated colors. The table in chapter 10 shows the highest saturations for red at $0.63\mu\text{m}$, for green at $0.54\mu\text{m}$ and for blue at $0.47\mu\text{m}$.

16. Hoffmann's prism experiment

A slide projector is used as light source. The slide is a metal plate with a thin horizontal slit. The cylinder with the prism can be rotated. The spectrum bar appears on an adjustable glass plate which is ground on one side. The mechanical system can be folded and stored together with the projector in a suitcase. A student's project by Mr. Burhan Primanintyo.

Best view zoom 200% at 72dpi.



17. Reverse problem and inverse appearance

In September 2012 the author received a message by Dr. Stephen R.J. Williams, a UK-based Geoscientist:

The images in your chapter 12 seem to explain how the fringes are produced. My interpretation (see attached illustration) of your diagram indicates that observer will see blue fringes at top of aperture image and red/yellow fringes at bottom of aperture image. But when I look through a real prism I see blue fringes at the bottom of the aperture image and red/yellow fringes at the top. This is opposite to that in the diagram.

Dr. Williams is right. Later he provided the link [26], which is of high value. But this report does not explain the problem in a convincing manner, does not explain what is called below the *inverse appearance phenomenon*.

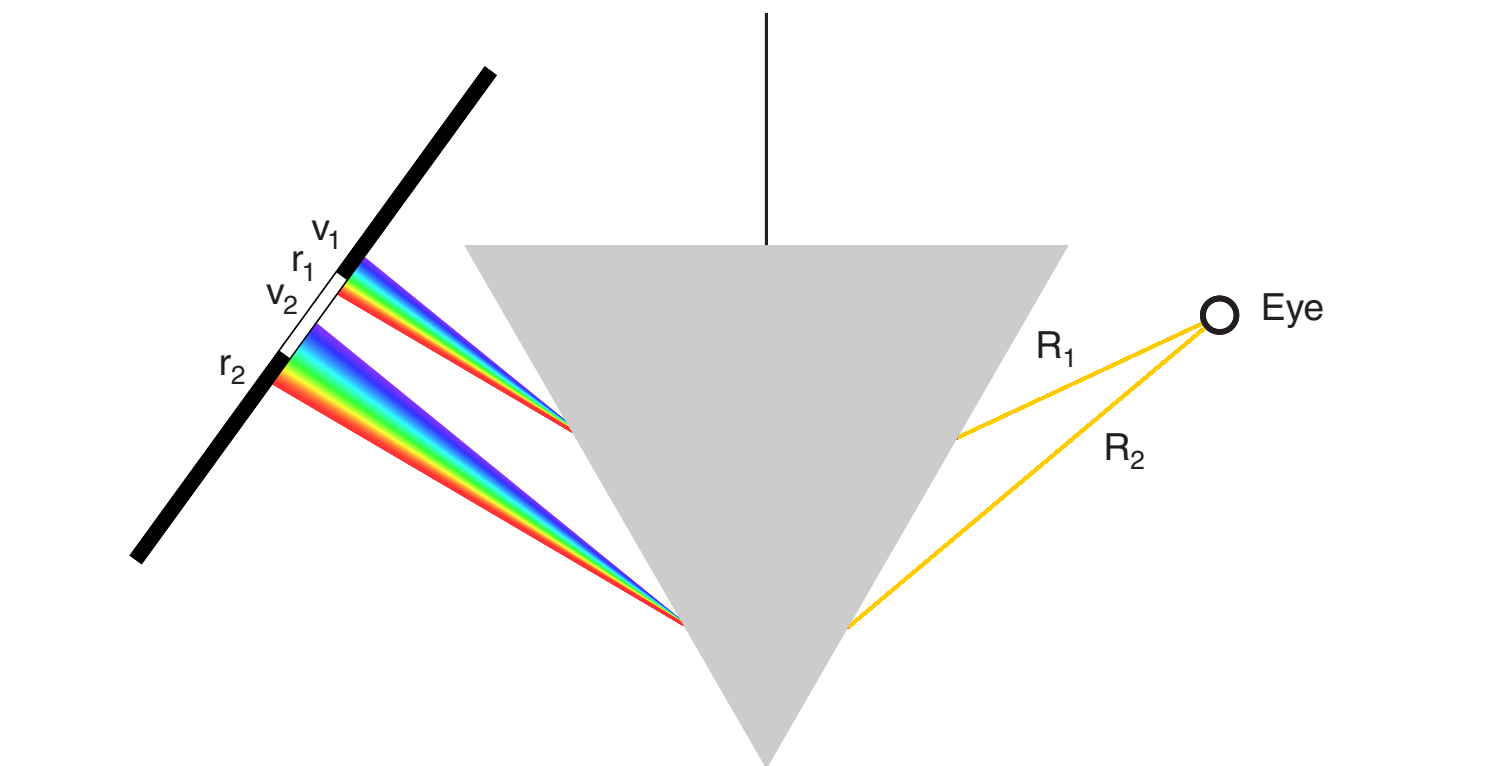
Not even Goethe himself had described the discrepancy between his drawing *Fünfte Tafel* in chapter 14 and his view through the prism. The drawing reproduces more or less Newton's approach (with some errors though, concerning the angles of refraction). It shows the color bands on a screen, similar to all the other illustrations here in this document.

Looking through a prism is in [26] called the *reverse problem* - exactly Goethe's approach, but not in his drawing. The inversion, as observed by Dr. Williams, is here called *inverse appearance phenomenon*.

The explanation follows Dr. Williams' idea with minor modifications. At the left we have a black card board with a white rectangle, maybe like one of those which Goethe had used [27].

One eye of the observer is shown at the right. We are interpreting two rays R_1 and R_2 . Ray R_1 could be generated by a violet ray v_1 , a red ray r_1 or any other ray between (in modern nomenclature: orange, yellow, green, cyan, blue). Now we see that blue, cyan and green actually cannot be generated, because the card board is black at these positions. Therefore only red and yellow contributions remain and that is the color of the fringe. For ray R_2 the situation is similar, but violet, blue and cyan remain. Dr. Williams emphasizes that green hardly appears at edges, opposed to fully developed rainbow spectra for narrow apertures or rectangles. This can be confirmed by experiments using the *Goethe Kit* [27].

Somewhat simplified: the top fringe is red/yellow, the bottom fringe blue/cyan. This is indeed an inverse appearance, compared to the projection of rays through a prism onto a screen, as simulated in chapter 12.



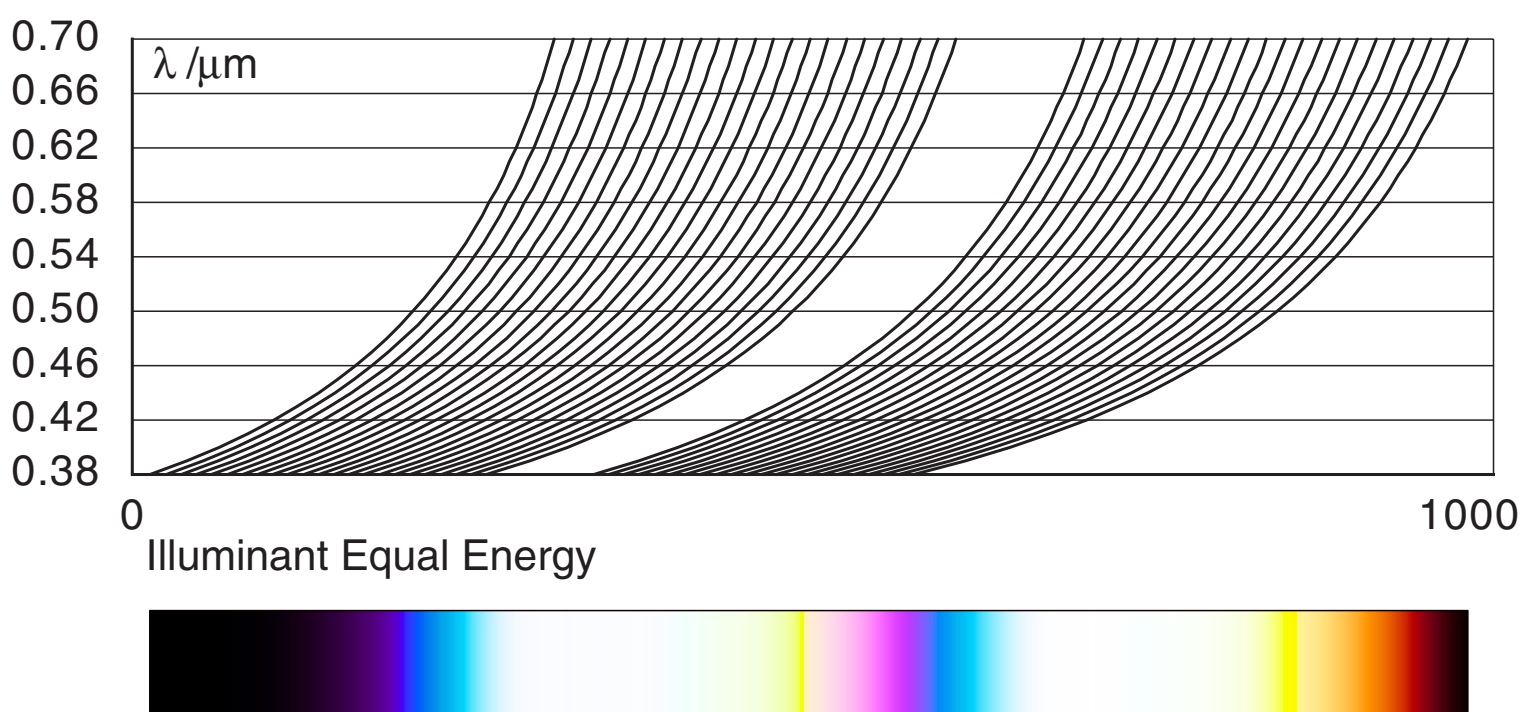
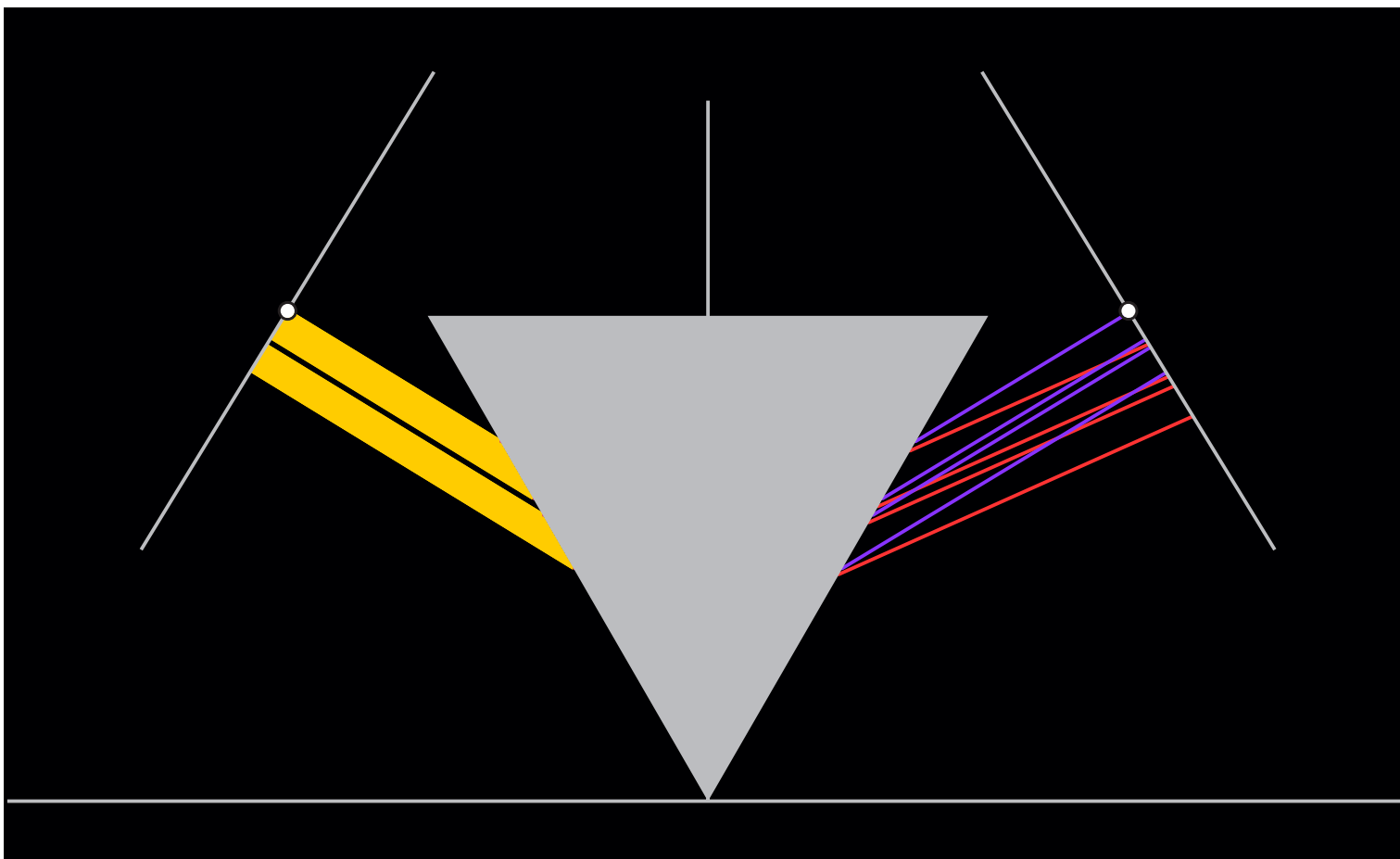
18.1 The colors of darkness

In July 2015 the author found in a newspaper an interesting article by Michael Jäger [28] about a new book by Olaf Müller, concerning Goethe's investigations [29]. There is an associated website by Olaf Müller [30].

M.Jäger says:

„In der Mitte seines Spektrums sieht Goethe die Farbe Purpur, die bei Newton nicht vorkommt. Bei Goethe kommt die Farbe Grün nicht vor. Müller kann nachweisen, daß bis jetzt alle Versuche, das Ergebnis des goetheschen Experiments “orthodox”, also newtonisch wegzu erklären, gescheitert sind. Gewiss kann man das Purpur als komplizierte Überlagerung der Farben des newtonschen Spektrums interpretieren...”

This interpretation is fairly simple: a black gap (darkness) in the generating object light band is mapped behind the prism onto the screen not as black but as a pseudo spectrum which contains indeed purple as a mixture of red and blue. (cont. next page)



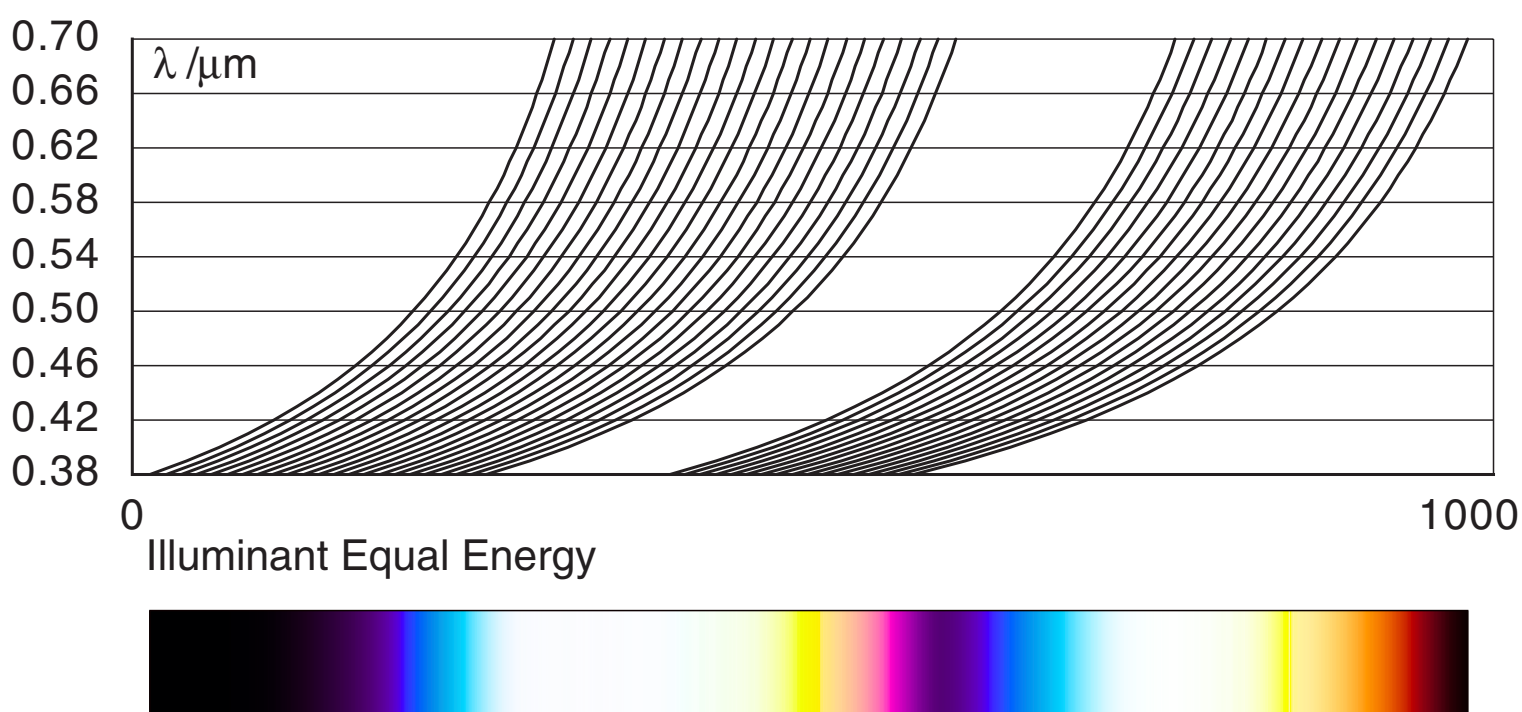
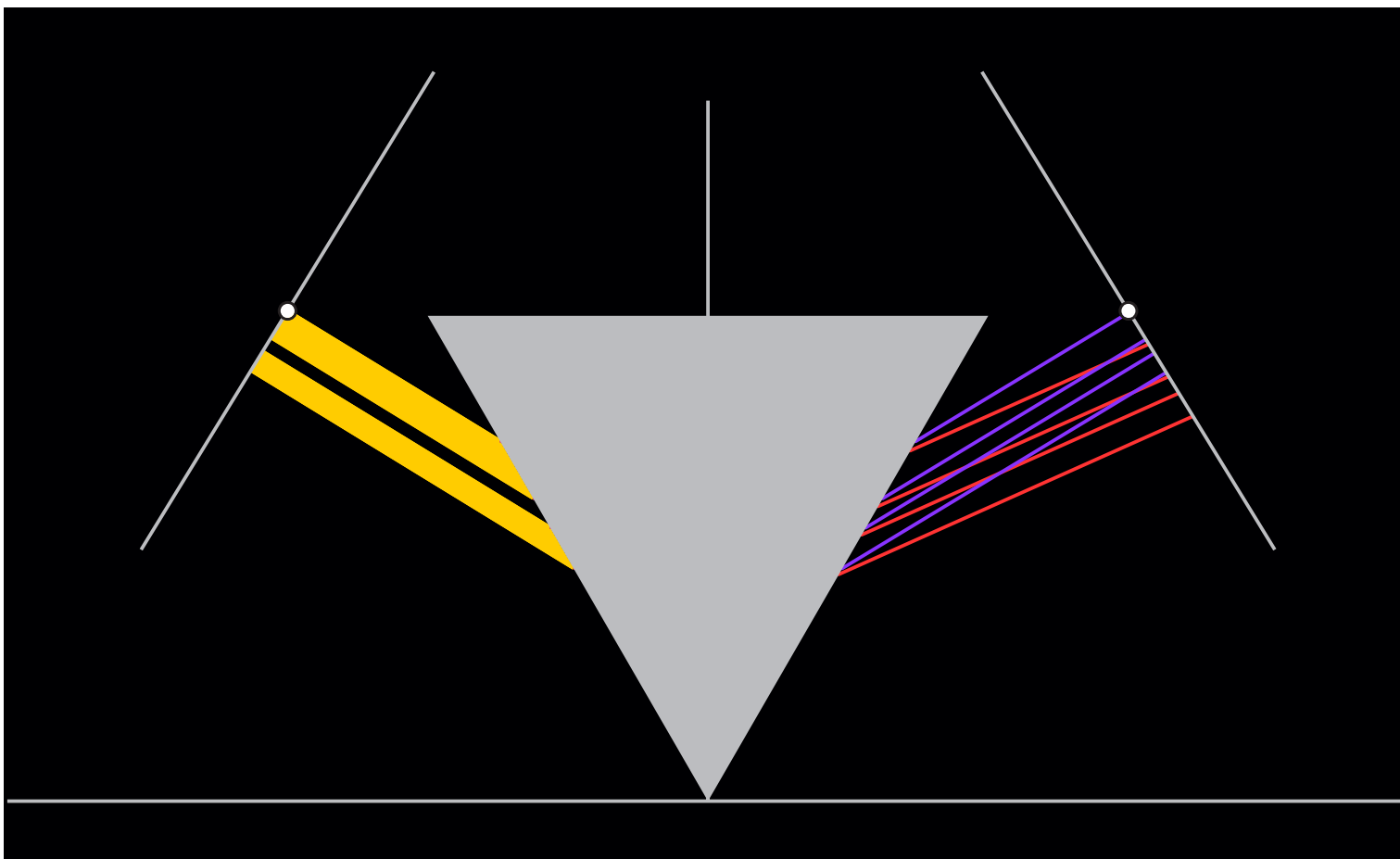
18.2 The colors of darkness

This test illustrates Jäger's *Dunkelstrahlen* [28], which refer perhaps to Goethe's remark in his *Confession des Verfassers* [2],[32]: „Eine schwarze Scheibe auf hellem Grund macht aber auch ein farbiges und gewissermaßen noch prächtigeres Gespenst. Wenn sich dort das Licht in so vielerlei Farben auflös't, sagte ich zu mir selbst: so müßte ja hier auch die Finsterniß als in Farben aufgelös't angesehen werden.“

This visualization is highly artificial. If the geometry and other parameters are modified, then the result will be different, see below. This 'verification' of Goethe's theory is widely a fake.

Referring to [30], Müller's website: The darkness does not create the primaries cyan, magenta and yellow for the subtractive color mixture. It's just an overlay of spectrum parts, of course generated by light and not by darkness, somewhat optimized for the result purple=magenta. Cyan and yellow belong anyway to the spectral colors, but the color purple doesn't.

A critical interpretation of Olaf Müller's book can be found in [31].



19. A natural rainbow

This image shows a natural rainbow. The saturation in the window was slightly enlarged in order to show two effects:

- 1) The rainbow colors contain the color cyan between green and blue, which is missing in Newton's descriptions.
- 2) Beyond blue one can see purple or violet. The existence of this color at the blue end of the spectrum (not at the red end) can be considered as a good reason for arranging the linear spectrum of visible colors on a circle. In fact we have two metameric purples: one at the blue end of the spectrum, therefore a spectral color, and the other by mixing blue and red.



Photo by courtesy of Norbert Kustos
Cropped and modified with permission

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Gernot Hoffmann
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