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Perceptually Optimized Grayscales

This document shows an attempt
to find transfer functions for
perceptually uniform Grayscales

Linear Transfer Function

Uncompensated Transfer Function

CIELab Conversion

Perceptual Optimization

Perceptually Optimized Grayscales

A Grayscale is defined by e.g. $n = 64$ stepwise increasing values L_{sk} in the range of $L_{sk} = 0.0 \dots 1.0$. For $n = 64$ the increment is $dL = 1/63$. We do not use $n = 256$, because then the differences between neighbours are not always perceivable.
The law is simply $L_{sk+1} = L_{sk} + dL$ for $k = 0, \dots, 63$, starting with $L_{s0} = 0$.
The color itself is primarily defined by $C_k = R_k = G_k = B_k = \text{Round}(255 \cdot L_{sk})$.
The rounding inaccuracy effect in color patches is avoided by dithering.
$C = 102.4$ is represented by different pixels, 60% of $C = 102$ and 40% of $C = 103$.
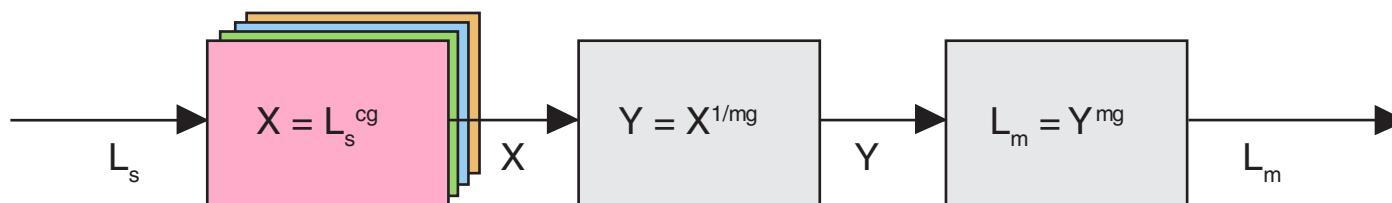
Since December 2007 all pages are lossless compressed (ZIP8).

L_s		Numerical data, linear light space, scene light
X	$= L_s{}^{cg}$	Perceptual Correction
Y	$= X^{1/mg}$	Monitor Gamma Compensation
L_m	$= Y^{mg} = L_s{}^{cg}$	Monitor Luminance

Important Note

CIELab Lightness (index c for CIELab variables)

If $Y_c > 0.008856$ Then $L_c^* = 1.16 \cdot Y_c^{1/3} - 0.16$ Else $L_c^* = 9.033 \cdot Y_c$

If $L_c^* > 0.008856 \cdot 9.033$ Then $Y_c = [(L_c^* + 0.16)/1.16]^3$ Else $Y_c = (1/9.033) \cdot L_c^*$

We use $L_s = L_c^*$ as input and $X = Y_c$ as output of the first block. For simplicity we can say, this is a correction law with cg = 3.0 . For the test patches we use always the correct law, as above.
Of course the inverse monitor compensation has to be applied as well.

All examples are valid for Monitor-Gamma mg = 2.2 .

Once again: cg describes the total transfer function from linear data to monitor luminance
for an ideal calibrated monitor. CieLab is expected to show perceptually linear grayscales (wrong).

Case 1	cg = 1.0	Linear TransferFunction	Black background
Case 2			White background

Case 3	cg = 2.2	Uncompensated Transfer Function	Black background
Case 4			White background

Case 5	cg = 3.0	CieLab Mode	Black background
Case 6			White background

Case 7	cg = 1.54	Optimized Mode	Black background
Case 8			White background
Case 9			Gray background

Resume 1

L_s Luminance in a linear light space
L_m Luminance of ideal monitor

Case 1+2 (page 4+5). $L_m = L_s^{1.0}$
Linear Transfer Function looks too light
Linear doesn´t show differences at the light end.
Linear image processing is good, but an output correction is necessary

Case 3+4 (page 6+7). $L_m = L_s^{2.2}$
Uncompensated Transfer Function looks too dark at the dark end

Case 5+6 (page 8+9). $L_m = L_s^{3.0}$
CieLab Mode looks too dark
CieLab on white background shows eight nearly equal grays the dark end.
CieLab is bad

Case 7-9 (page 10-13). $L_m = L_s^{1.54}$
Optimized Mode shows reasonable resolutions at the dark and the light end
This is valid for black and white backgrounds

Accurate View:

Calibrated monitor
Gamma = 2.2

Text and line art smoothed
Images not smoothed

Zoom 200%

A computer graphic, made by a linear light model:

This compensation has to be done by calculations:

Without any compensation $c = 1.0$
With monitor compensation only $c = 1.0/2.2 = 0.4545$
Optimized compensation $c = 1.54/2.2 = 0.7$

A scan, as usual monitor compensated by 1/2.2:

This compensation has to be done by calculations:

With monitor compensation only $c = 1.0$
Optimized compensation $c = 1.54$

Further experiments showed some unexpected results:
The scanner menue indicated *2.2* , the necessary compensation was $c = 0.75$. Just the opposite result. It can be assumed, that the scanner message means something like *compensated for Gamma=2.2 monitors*, but not necessarily by the expected inverse law.

Nomenclature

Compensation
Modification of an image by a power function.
Executed in one direction without a reverse operation.
As shown in the signal flow diagram on page 2, the monitor Gamma=2.2 is always already corrected.

Gamma Working Space
This is not discussed here.
Linear light image data are distorted by a power function.
Calculations with these non-linear data are in principle wrong.
The reverse transformation is applied by the monitor.
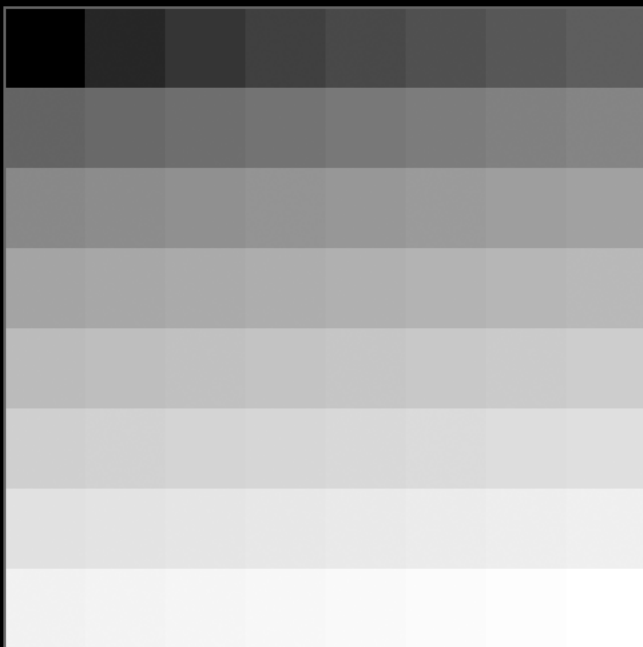
Case 1

Linear Transfer Function

Black background

$X = L_s^{1.0}$

$Y = X^{1/2.2}$

$L_m = Y^{2.2}$
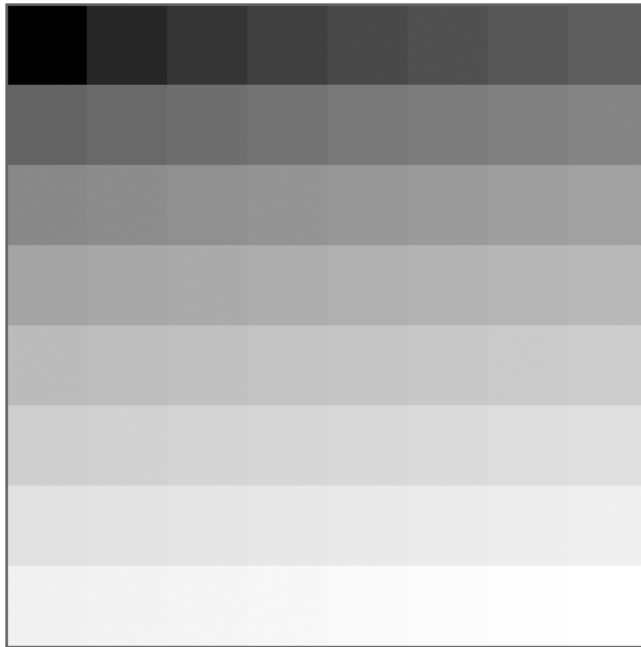
$L_m = L_s^{1.0}$

Case 2

Linear Transfer Function

White background

$X = L_s^{1.0}$

$Y = X^{1/2.2}$

$L_m = Y^{2.2}$
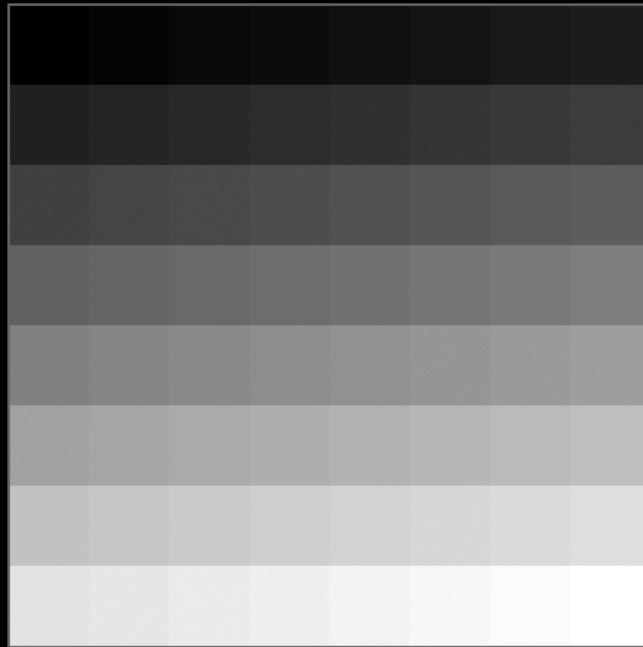
$L_m = L_s^{1.0}$

Case 3

Uncompensated Transfer Function

Black background

$X = L_s^{2.2}$

$Y = X^{1/2.2}$

$L_m = Y^{2.2}$
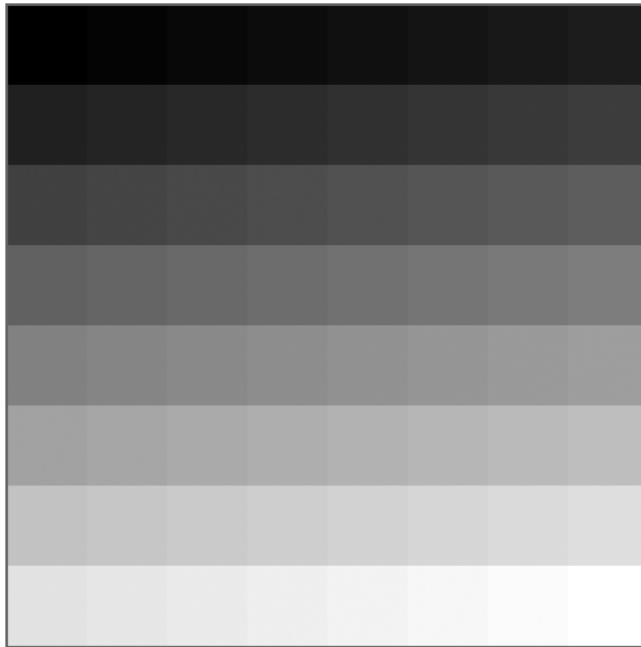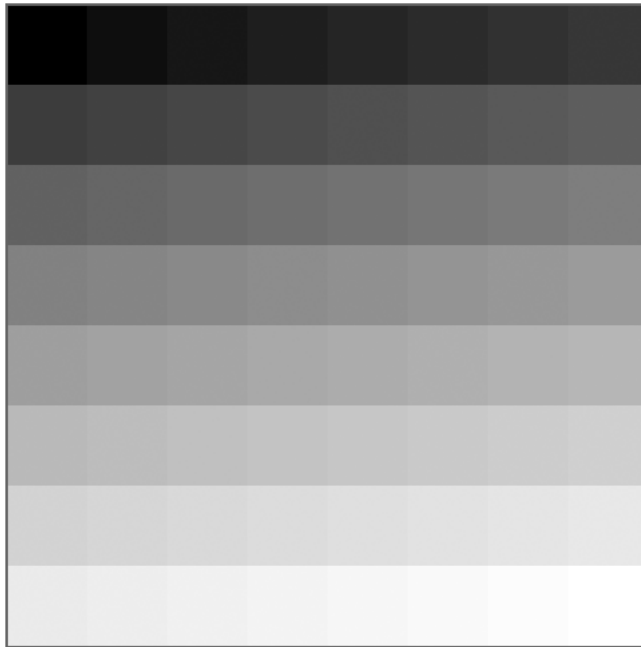
$L_m = L_s^{2.2}$

Case 4

Uncompensated Transfer Function

White background

$X = L_s^{2.2}$

$Y = X^{1/2.2}$

$L_m = Y^{2.2}$

$L_m = L_s^{2.2}$

Case 5
CIELab Mode
Black background

$$X = L_s^{3.0}$$

$$Y = X^{1/2.2}$$

$$L_m = Y^{2.2}$$

Case 6
CIELab Mode
White background

$$X = L_s^{3.0} \quad (\text{CieLab})$$
$$Y = X^{1/2.2}$$
$$L_m = Y^{2.2}$$
$$L_m = L_s^{3.0}$$
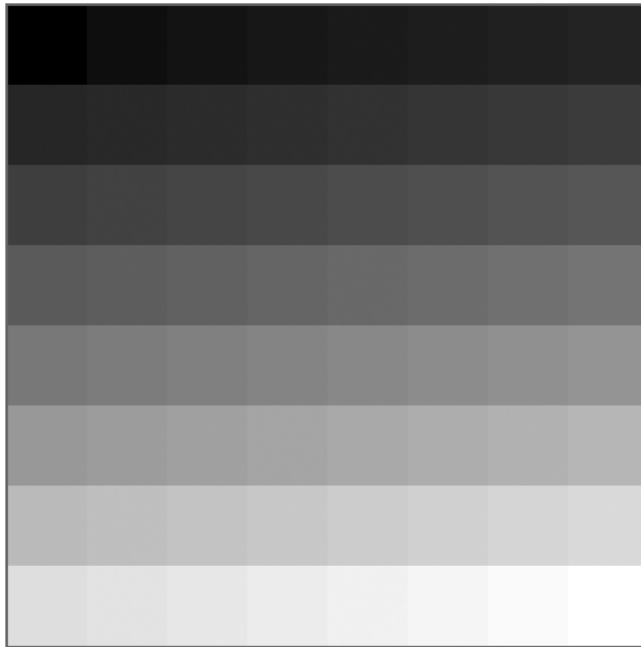
Case 7
Optimized Mode
Black background

$X = L_s^{1.54}$

$Y = X^{1/2.2}$
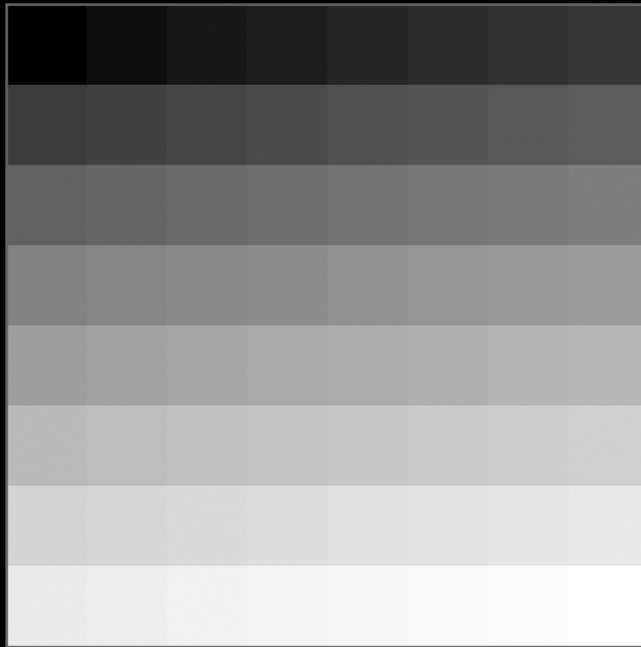
$L_m = Y^{2.2}$

$L_m = L_s^{1.54}$

Case 8
Optimized Mode
White background

$$X = L_s^{1.54}$$
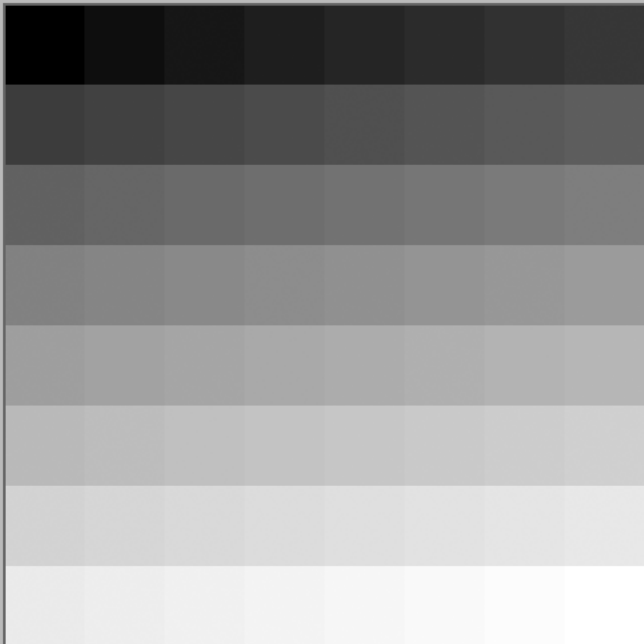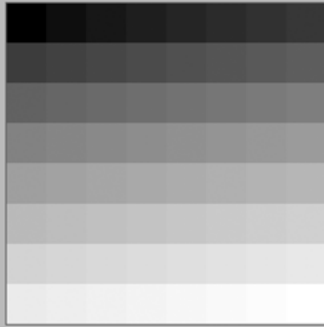$$Y = X^{1/2.2}$$
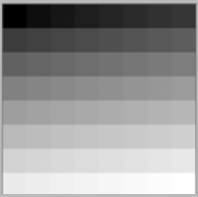$$L_m = Y^{2.2}$$
$$L_m = L_s^{1.54}$$

Case 9
Optimized Mode
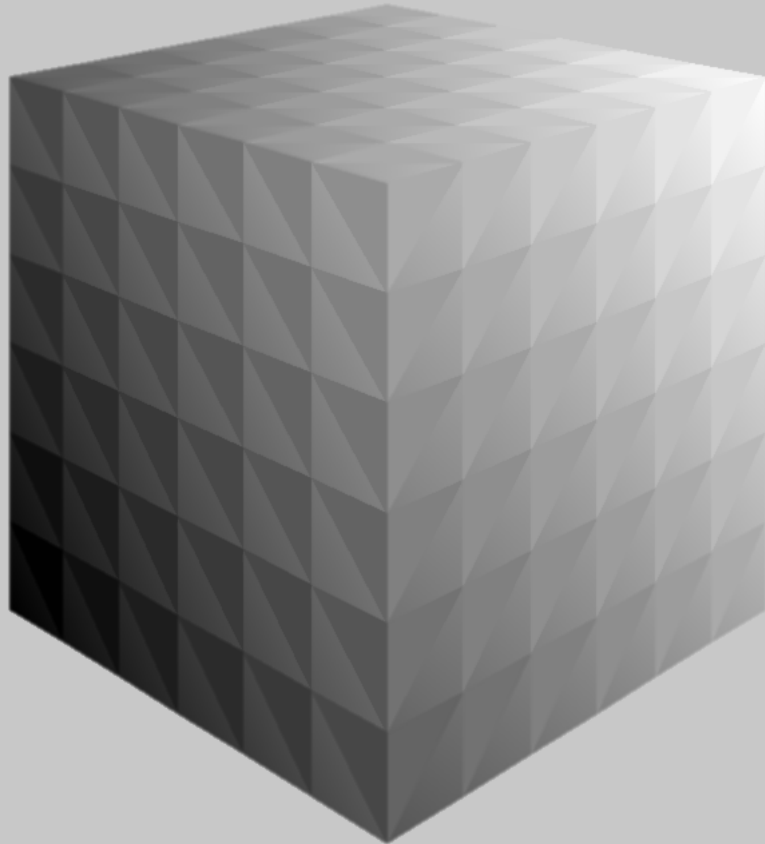Gray background (186)

$X = L_s^{1.54}$
$Y = X^{1/2.2}$
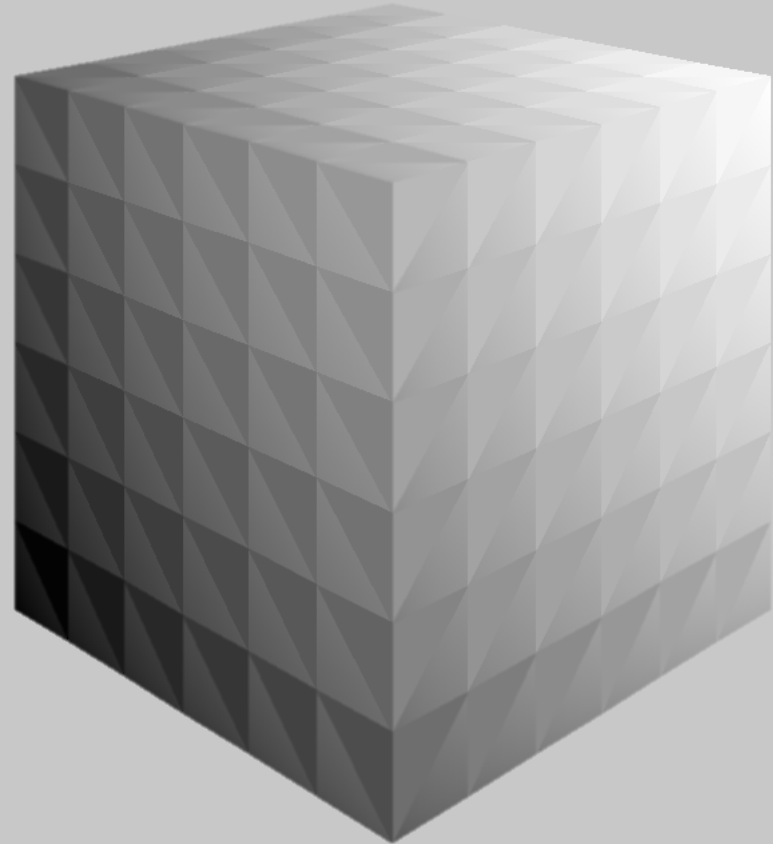$L_m = Y^{2.2}$
$L_m = L_s^{1.54}$

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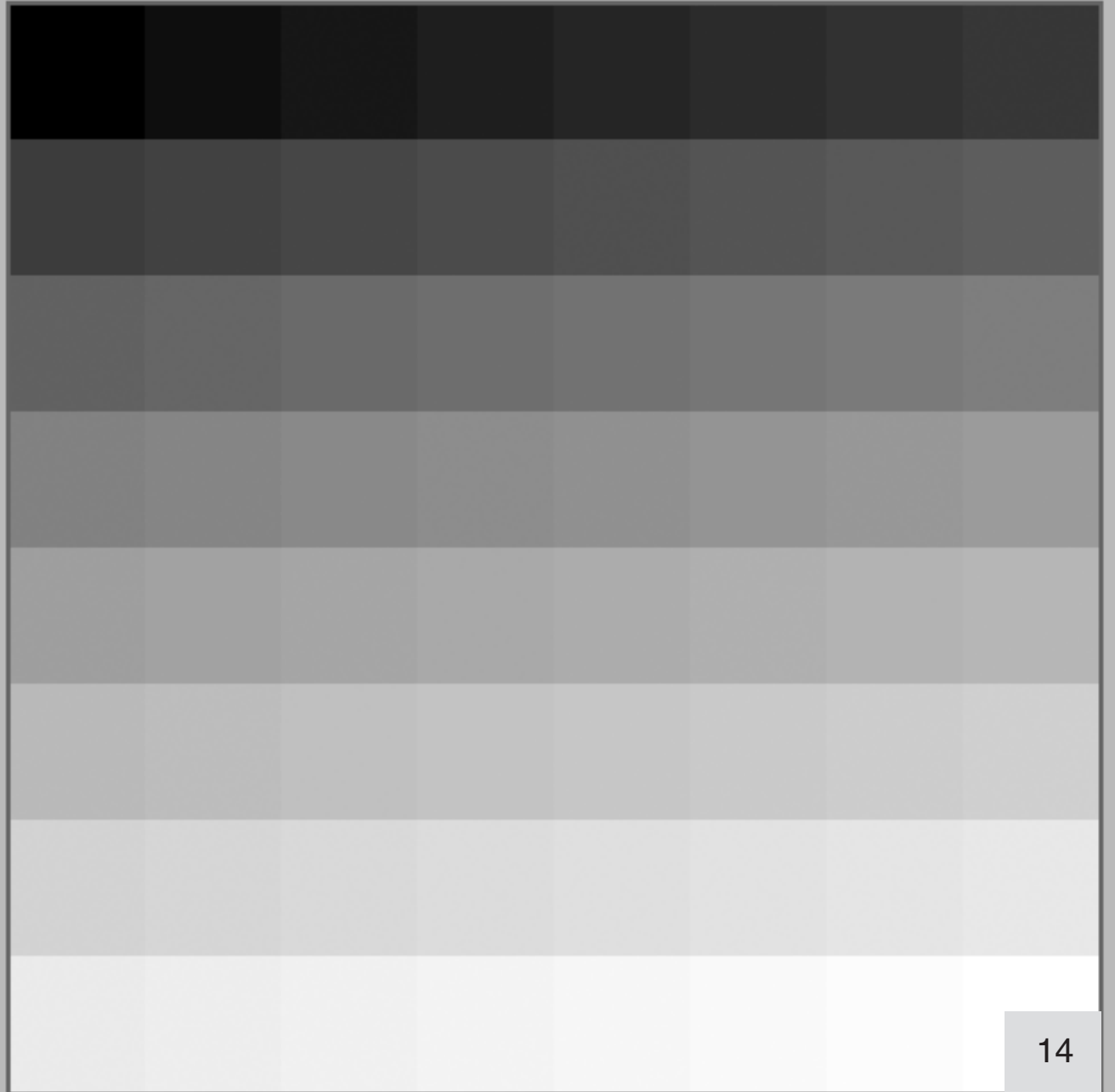
Color Cube with patterns
Grayscale conversion
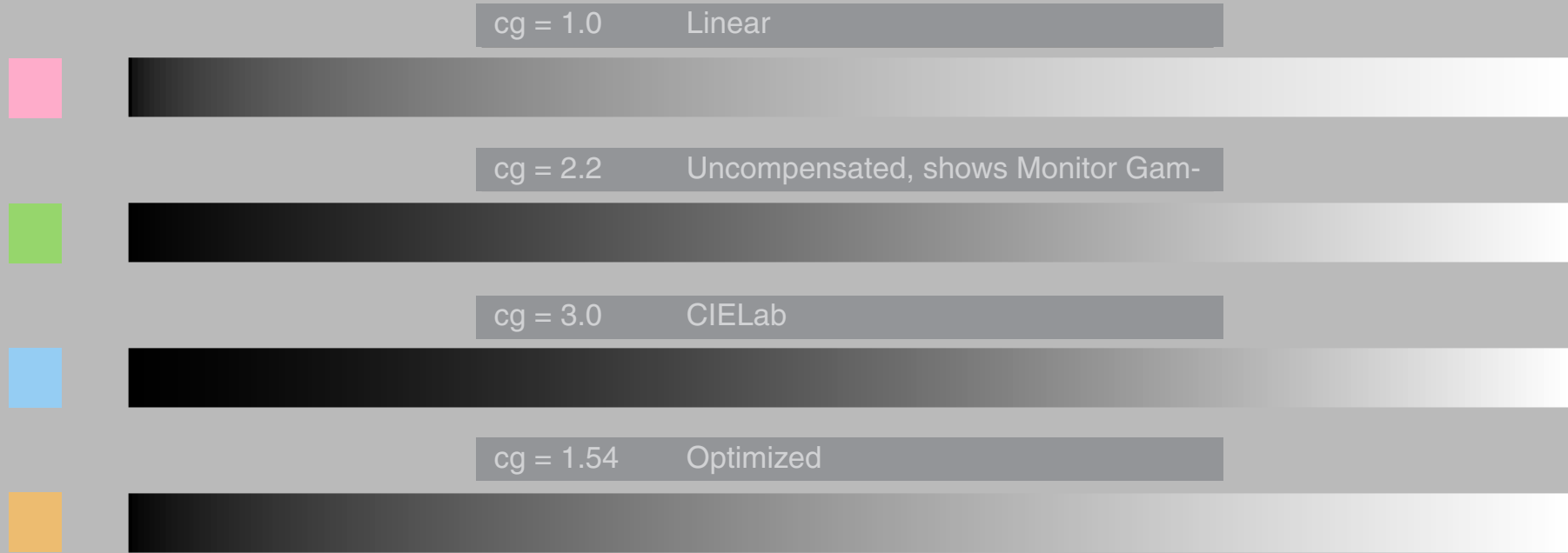Uncompensated $L_m = L_s^{2.2}$
Bad contrast for dark

Color Cube with patterns
Grayscale conversion
Optimized $L_m = L_s^{1.54}$
Better contrast for dark

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Final Test: Grayscales by a Different Program

$$L_m = L_s{}^{cg}$$

cg = 1.0   Linear

cg = 2.2   Uncompensated, shows Monitor Gam-

cg = 3.0   CIELab

cg = 1.54   Optimized

Uncompensated
Linear Gray
Distorted only by
Monitor Gamma
$L_m = L_s^{2.2}$

Pixel synchronized PDF
Please use zoom 200%
Height 512 pixels. Centered at Gray 127/128.

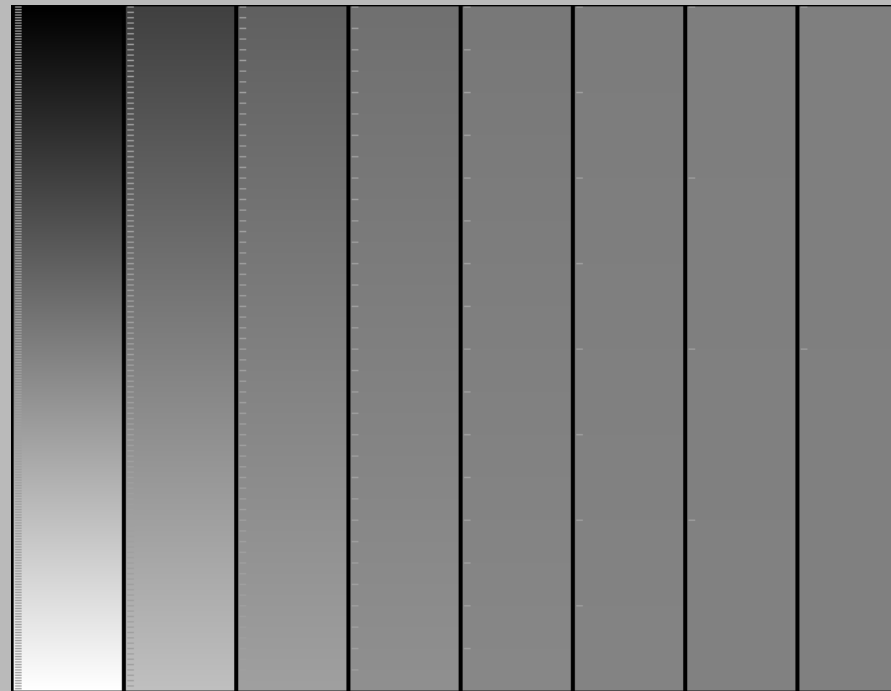Number of gray patches in one column

256   128   64   32   16   8   4   2
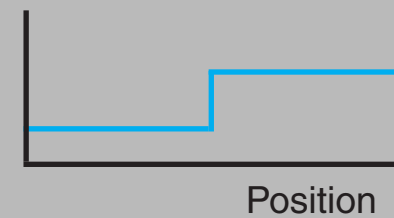
Mach Band Effect

Small transitions of the luminance can appear exaggerated as perceived lightness.
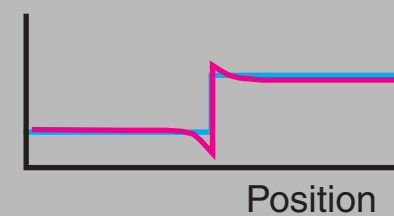This kind of differentiation is caused by the eye's neuronal net by 'lateral inhibition'.
This feature is probably helpful for finding contours in real life images.

Luminance

Position

Lightness

Position

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Uncompensated
Linear Gray
Distorted only by
Monitor Gamma
$L_m = L_s^{2.2}$

Pixel synchronized PDF
Please use zoom 200%
Height and width 512 pixels
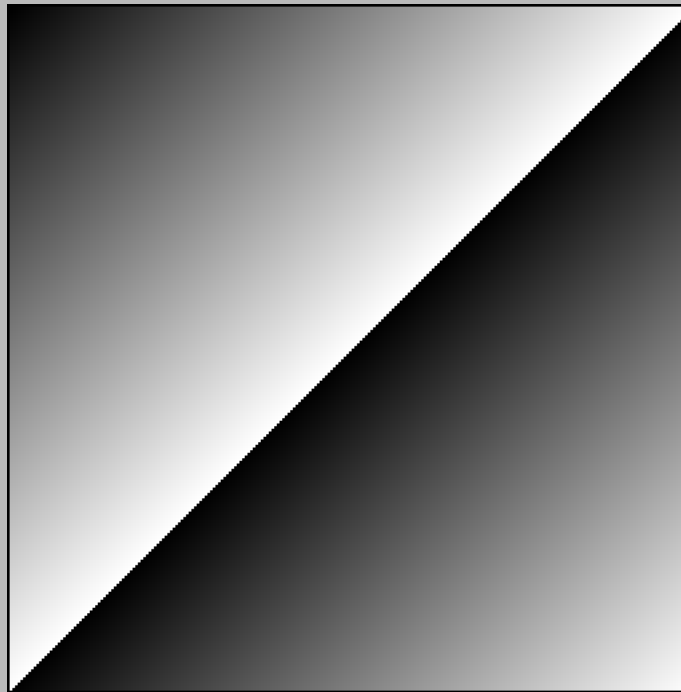
Lossless compression for all pages

Mach Band Effect

The effect is clearly visible on the pages 6 to 14, but not always visible on the previous page.
It depends on the monitor and the patch size.
A 256 step grayscale is usually not perceived banded.
The monitor grid may cause artifacts too.

Measured Brightness Perception

The diagrams are based on Fig.3 in [1]. The original doc shows absolute luminance from 0 to $111\,cd/m^2$ and perceived brightness by 10 values (not 11). The normalized diagrams below show relativ values from 0.0 to 1.0, stepsize 0.1. Therefore the corners of the polylines are not on the grid. The red curve is valid for patches on a flat medium gray background. The blue curve is valid for a similar test in a simulated threedimensional scene. The green curves were added by G.Hoffmann:

In the left diagram a power function with exponent 1/1.54. In the right diagram power functions for 1/1.0 to 1/2.2 for steps 0.2.
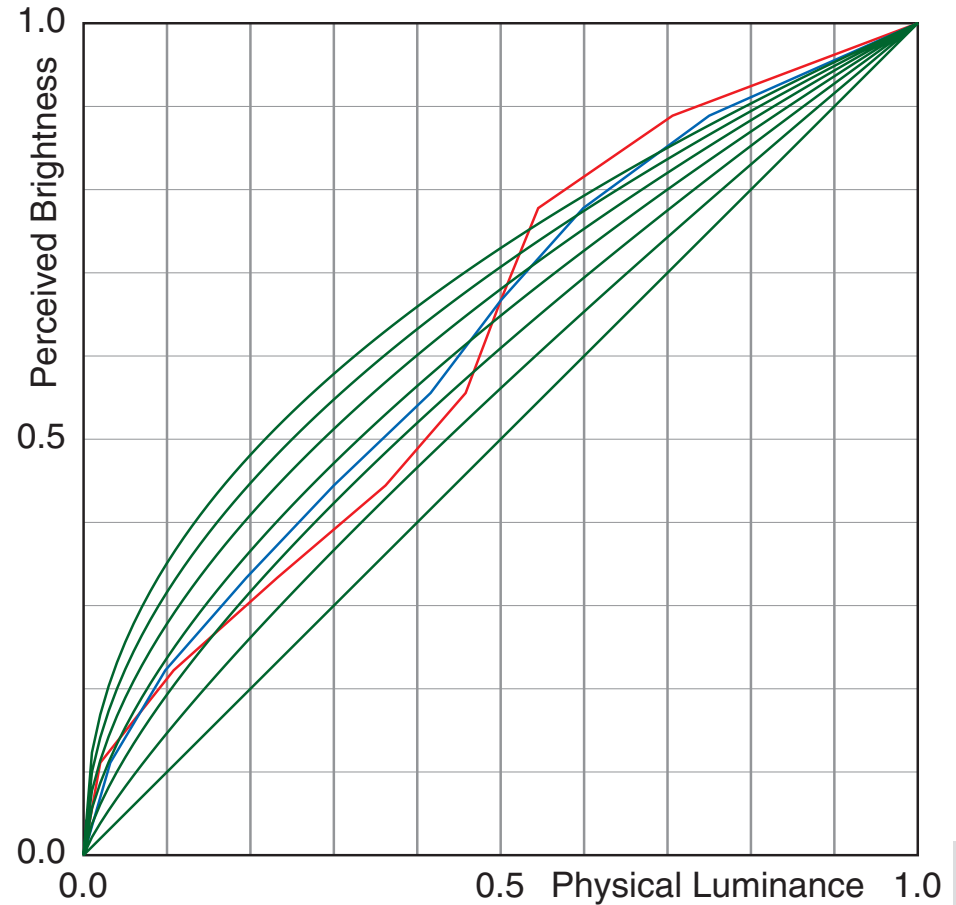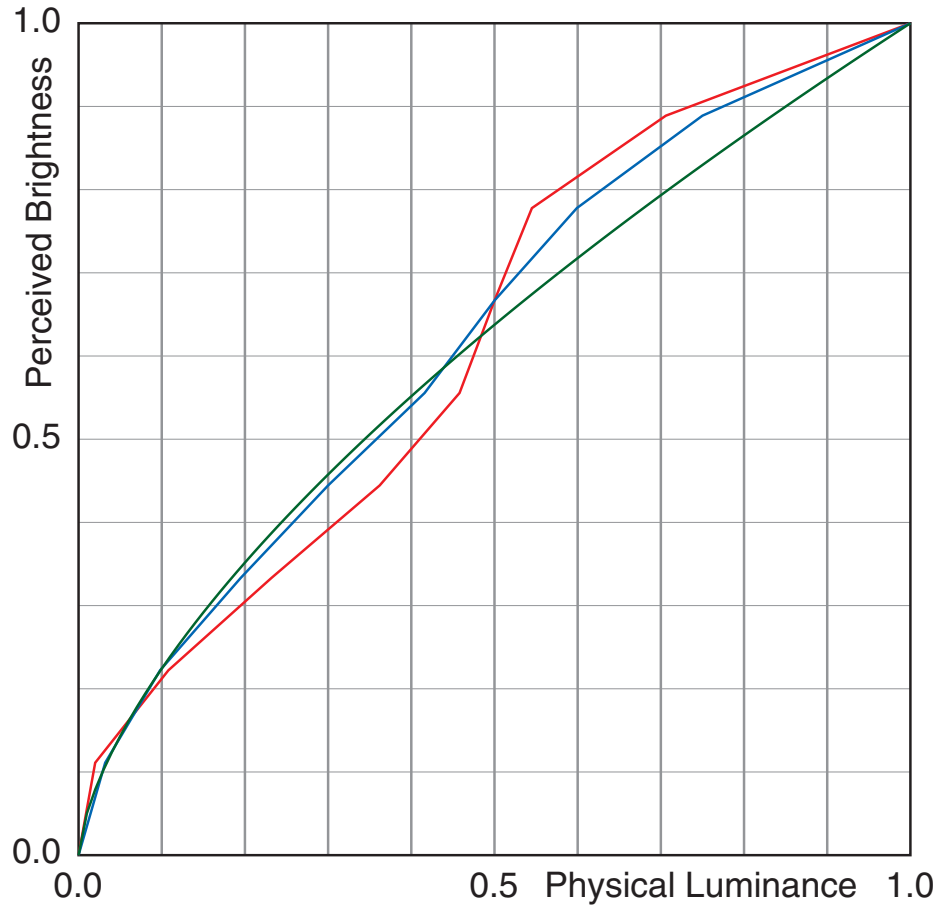
References

[1]   Surajit Nundy + Dale Purves
       A probabilistic explanation of brightness scaling
       14482-14487 / PNAS / October 29, 2002 / vol.99 / no.22
       http://www.pnas.org/cgi/doi/10.1073/pnas.172520399

[2]   Gernot Hoffmann
       The Gamma Question
       http://docs-hoffmann.de/gamquest18102001.pdf
       (more references)

       This doc:
       http://docs-hoffmann.de/optigray06102001.pdf