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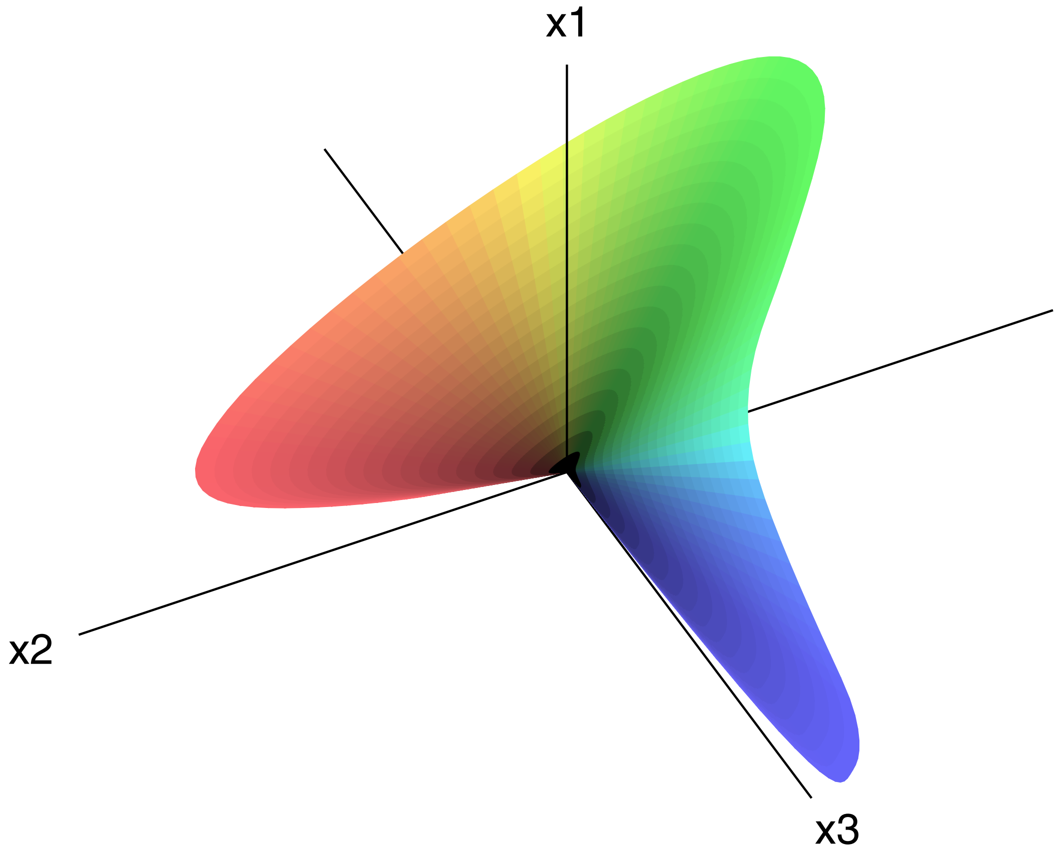
Locus of Unit Monochromats



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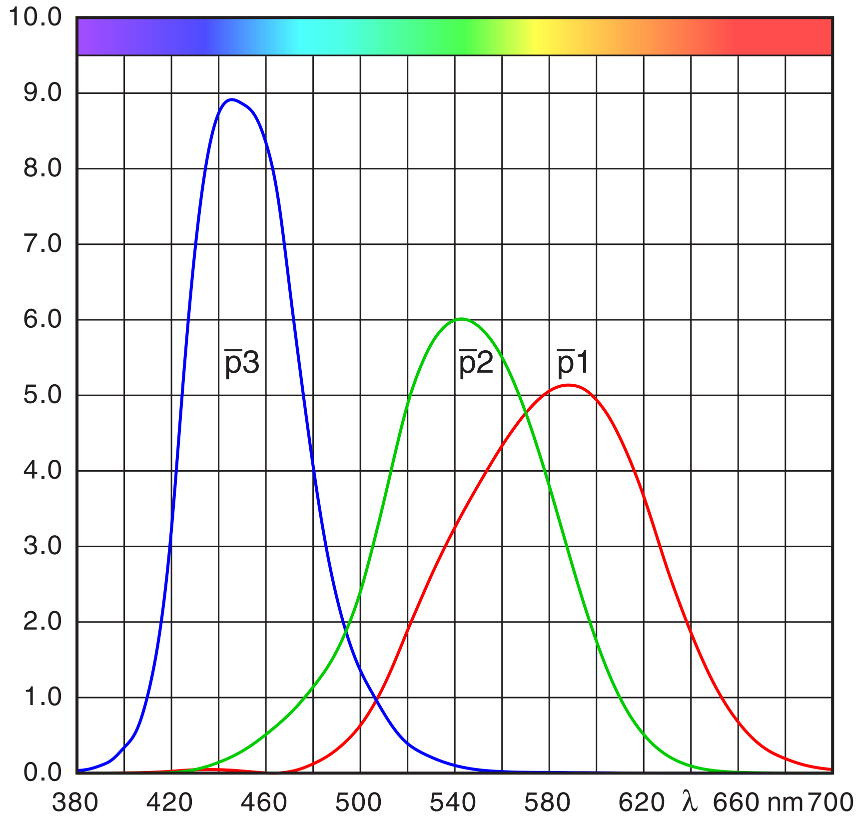
1. Introduction

This document does not contain much more than some illustrations and a few comments, based on several publications by James A. Worthey, especially [18].

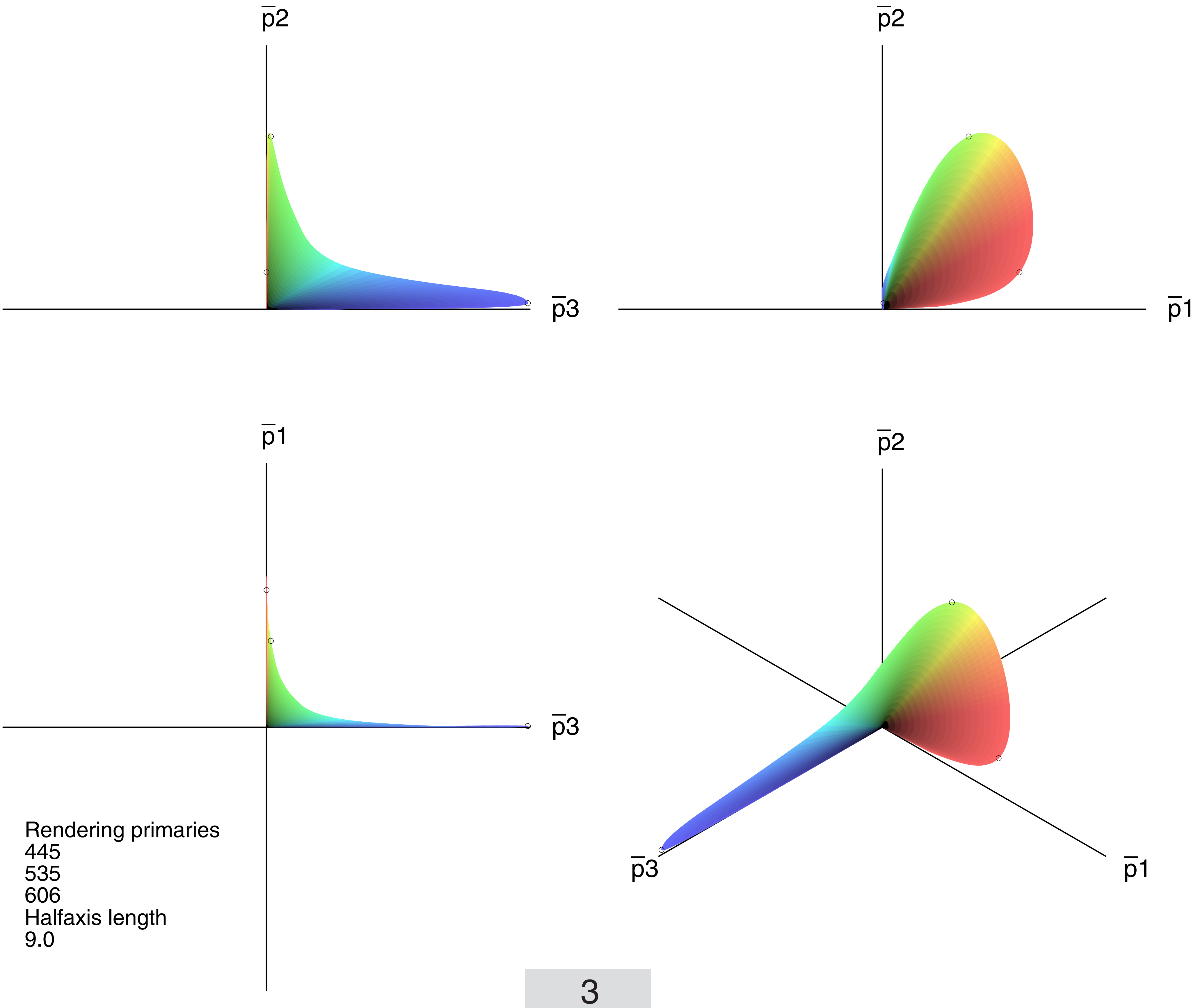
The illustrations show the threedimensional curves for cone response sensitivity functions and the color-matching function. Such a curve for a color-matching function is called the locus of unit monochromats (introduced by J.A.Worthey, mainly for his orthonormal functions, and based on older references as well). Connecting the curve points (varying wavelength) by straight lines with the origin, we get surfaces.

The different shapes are consequently shown by three orthographic views and one isometric view. This simplifies the comparison.

2. Cone Response Sensitivities

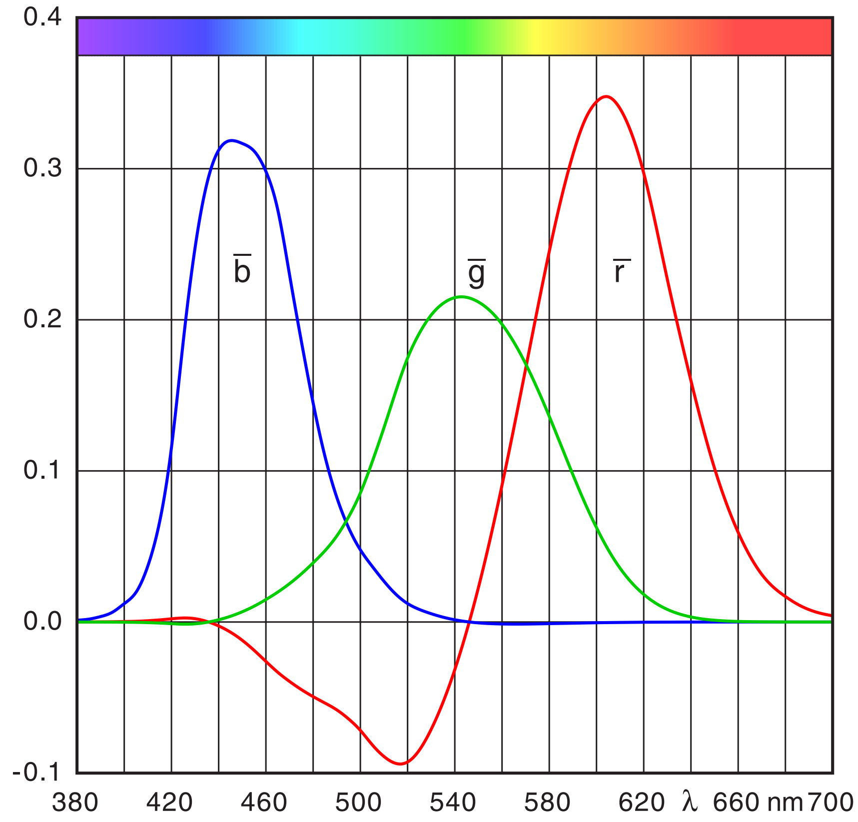


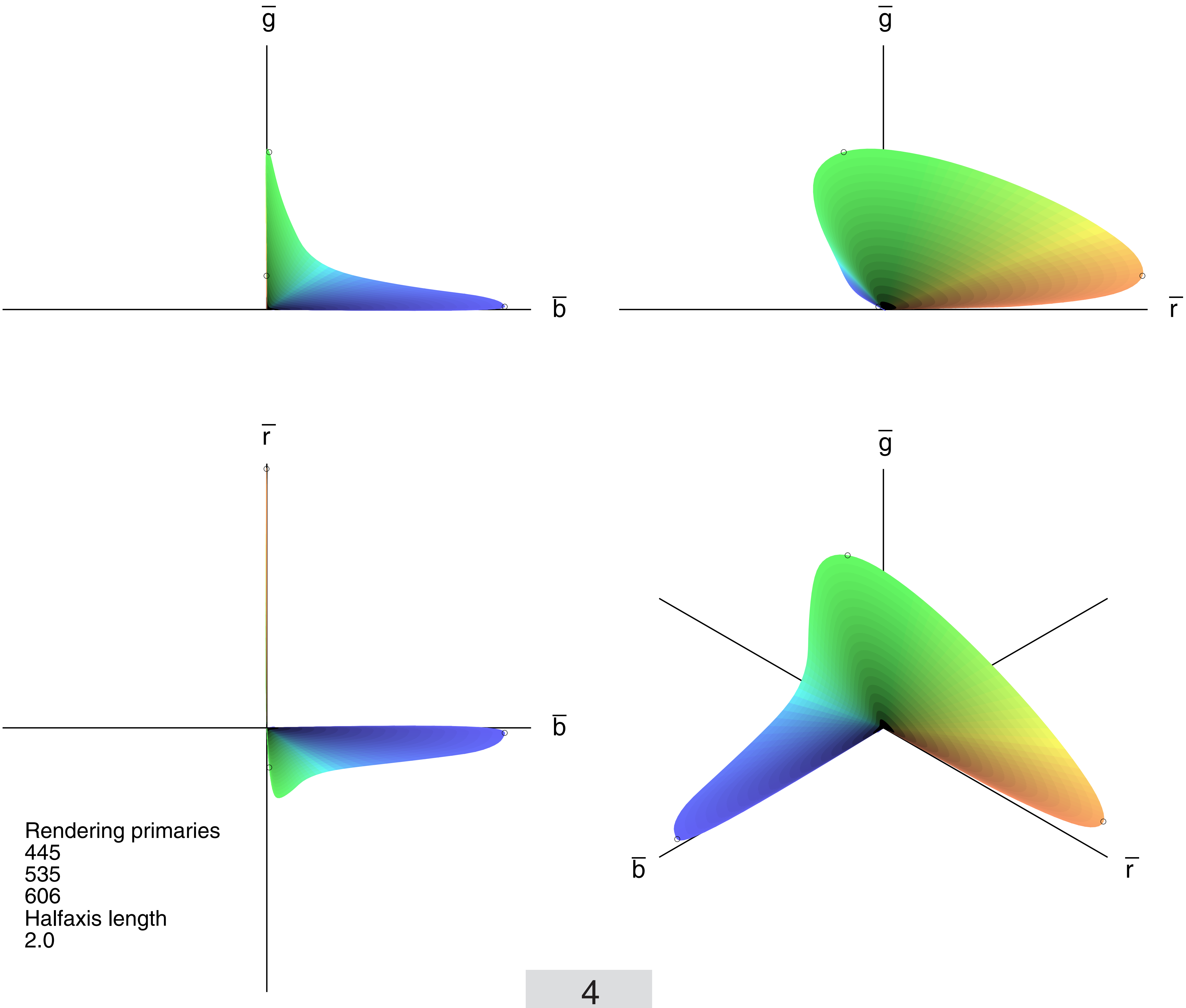
$$\begin{bmatrix} \bar{p}_1 \\ \bar{p}_2 \\ \bar{p}_3 \end{bmatrix} = \begin{bmatrix} 3.1956 & 2.4478 & -0.6434 \\ -2.5455 & 7.0492 & 0.4963 \\ 0.0000 & 0.0000 & 5.0000 \end{bmatrix} \begin{bmatrix} \bar{x} \\ \bar{y} \\ \bar{z} \end{bmatrix}$$



3. CIE RGB Color-Matching Functions

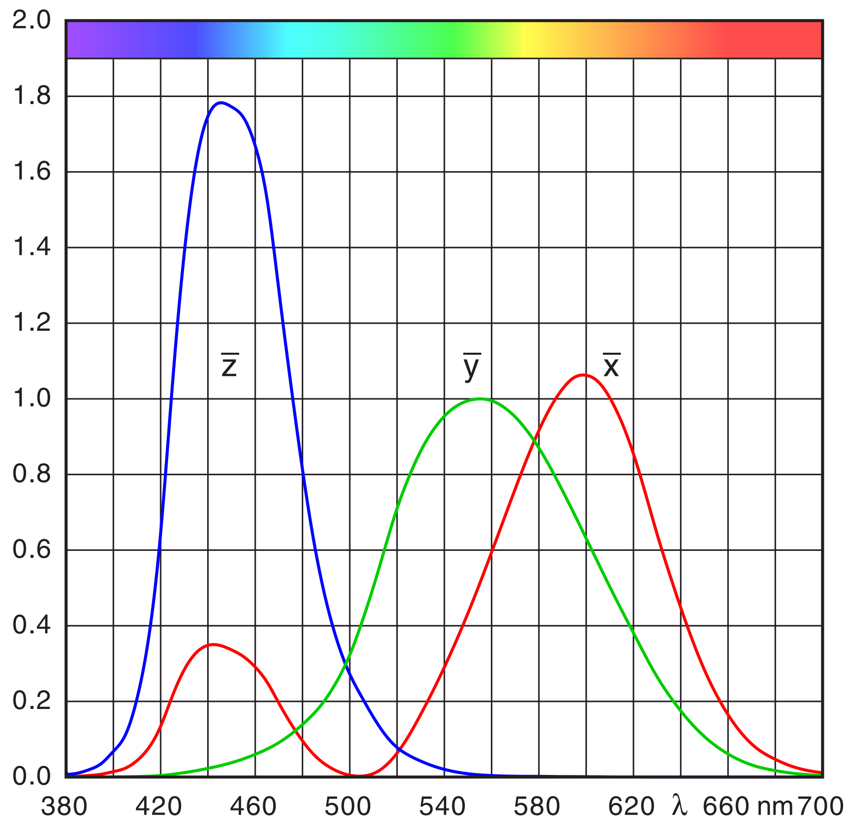
$$\begin{bmatrix} \bar{r} \\ \bar{g} \\ \bar{b} \end{bmatrix} = \begin{bmatrix} 2.3647 & -0.8966 & -0.4681 \\ -0.5152 & 1.4264 & 0.0887 \\ 0.0052 & -0.0144 & 1.0092 \end{bmatrix} \begin{bmatrix} \bar{x} \\ \bar{y} \\ \bar{z} \end{bmatrix}$$

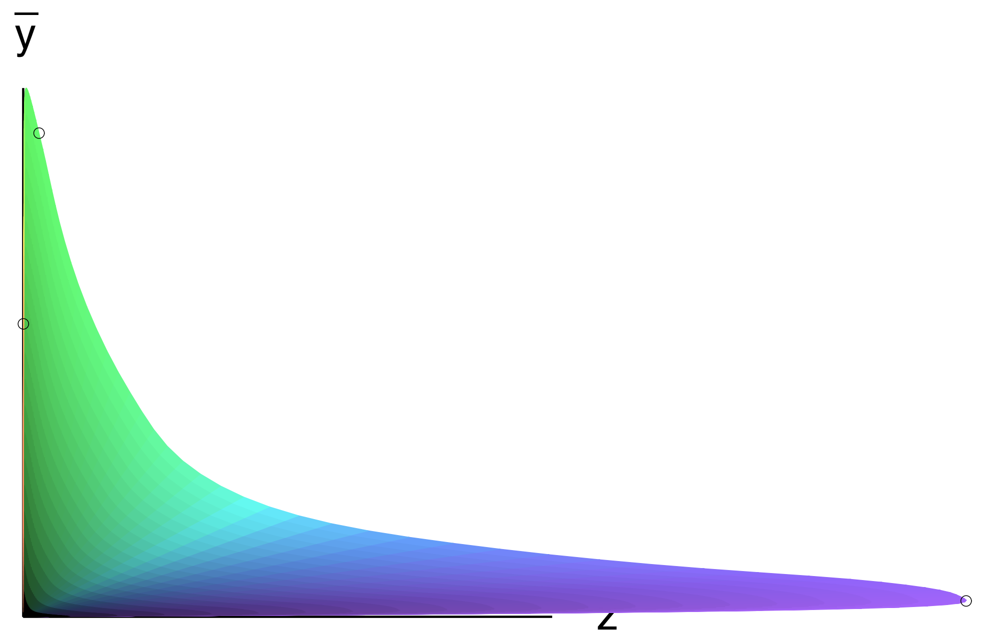


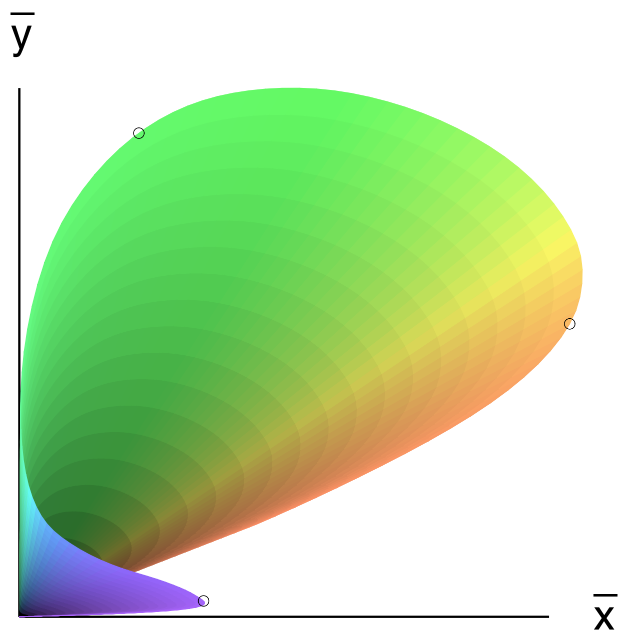


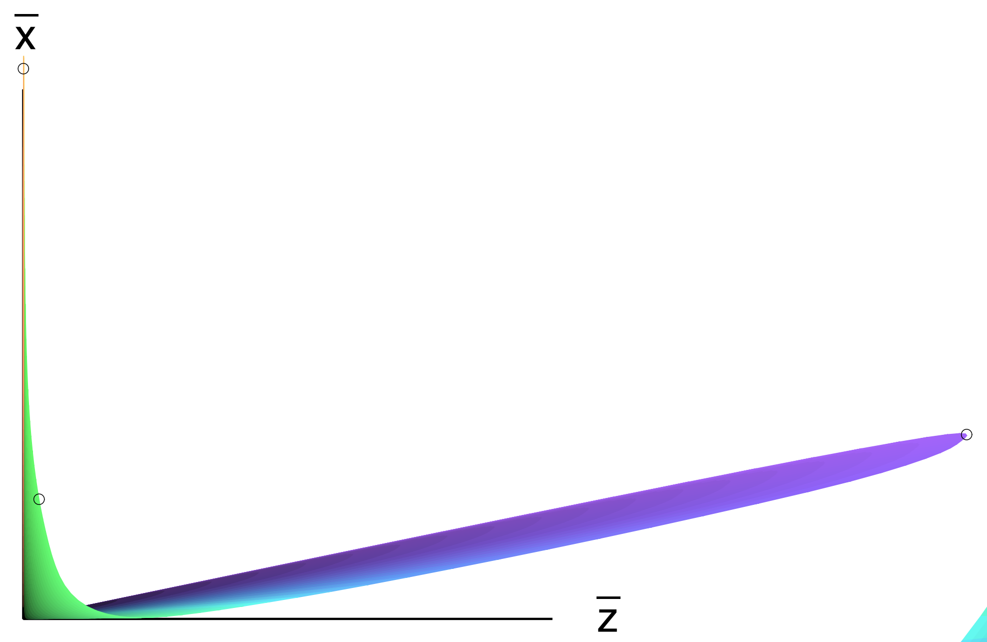
Rendering primaries
445
535
606
Halfaxis length
2.0

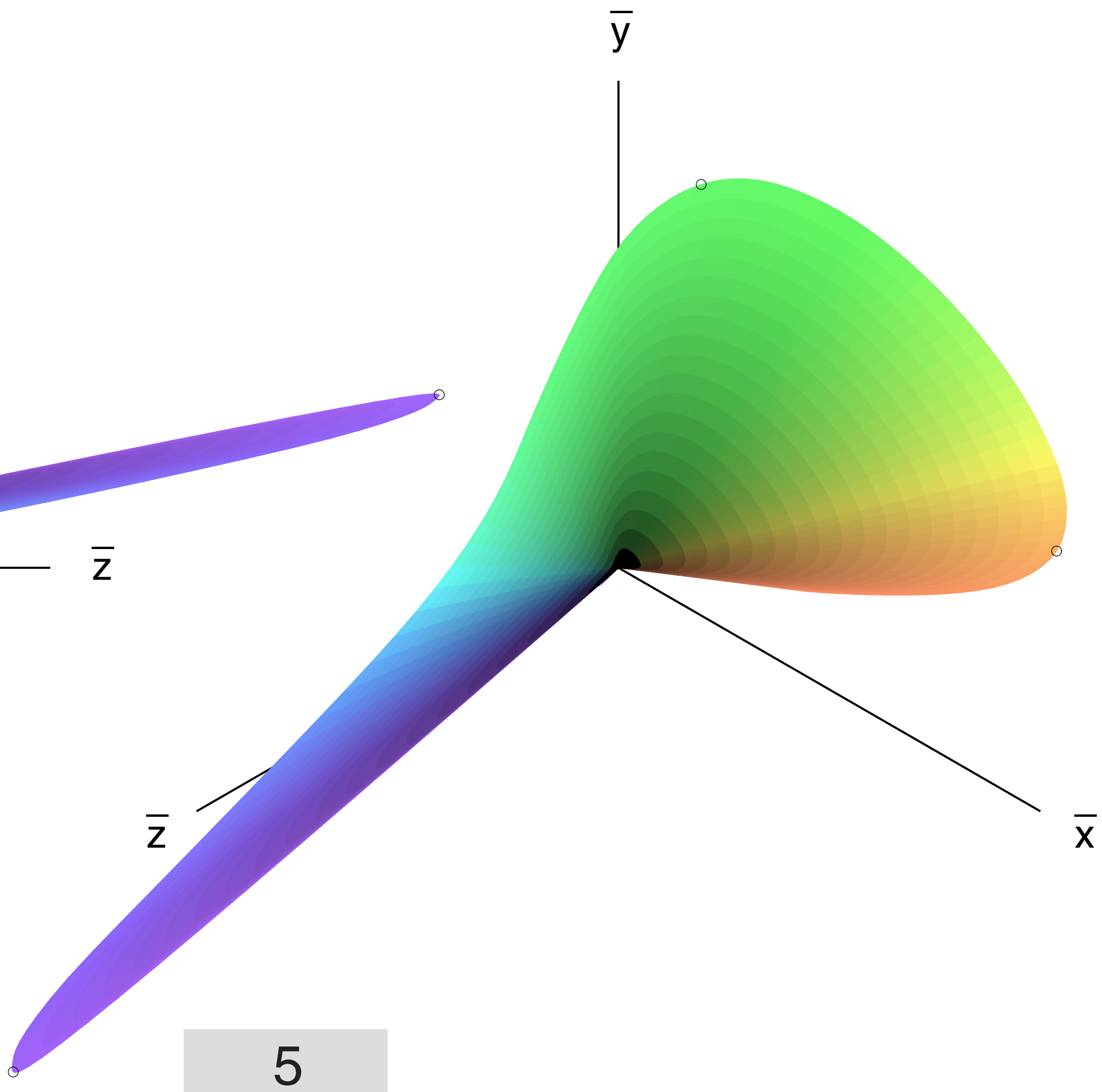
4. CIE XYZ Color-Matching Functions









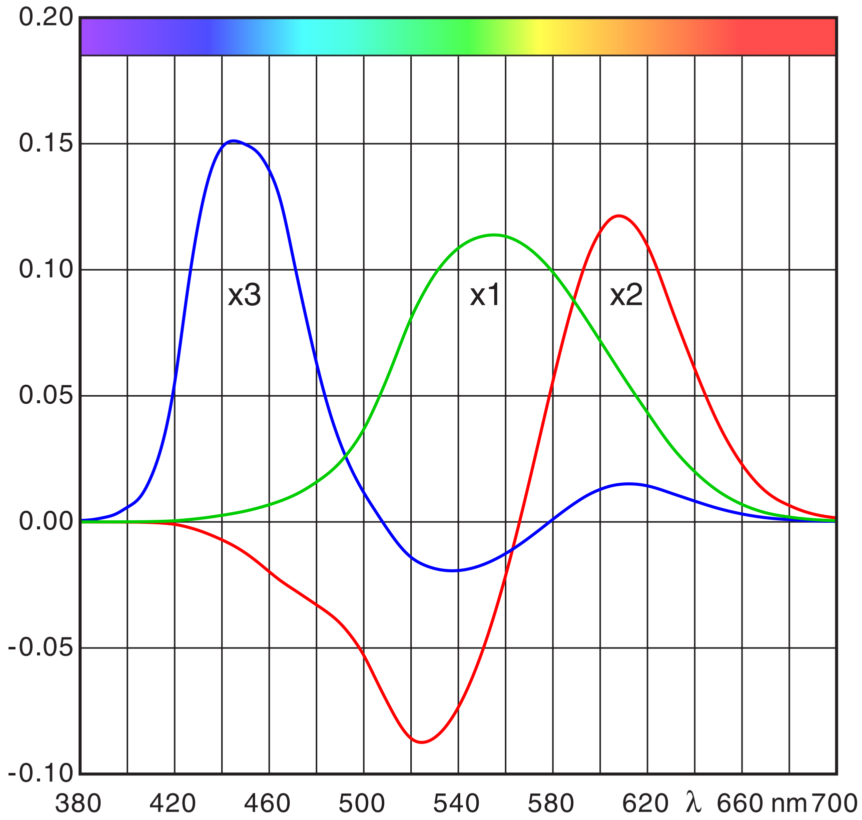


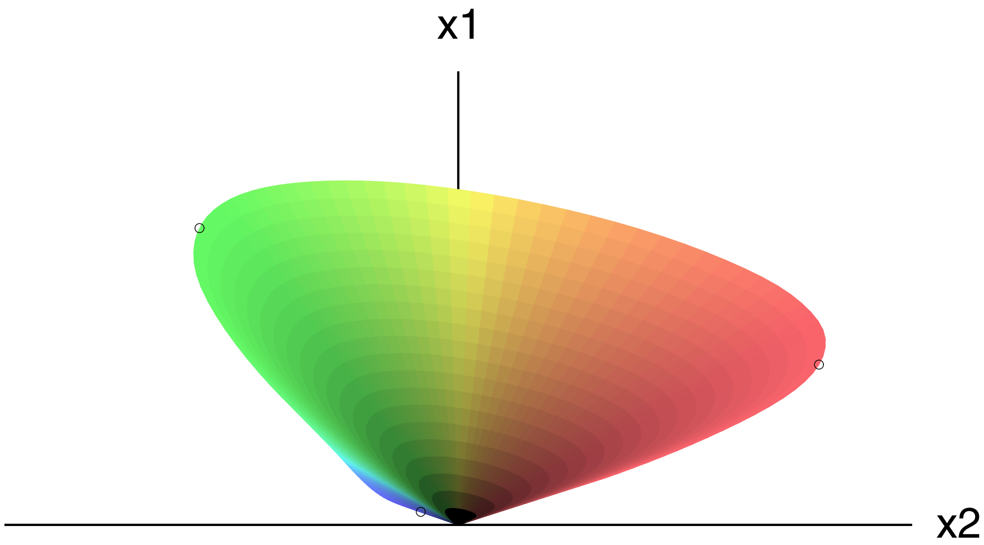
Rendering primaries
445
535
606
Halfaxis length
1.0

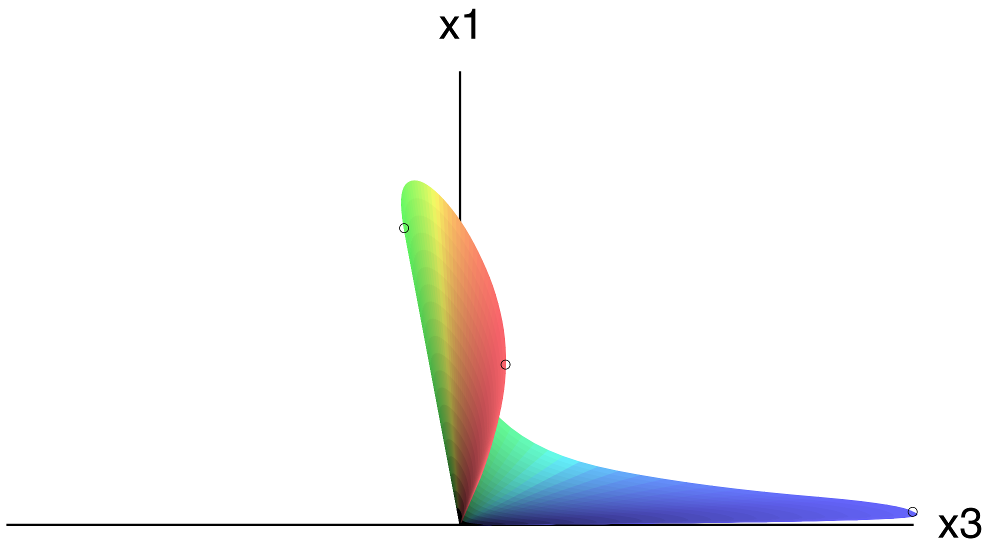
5.1 Worthey's Orthonormal Color-Matching Functions

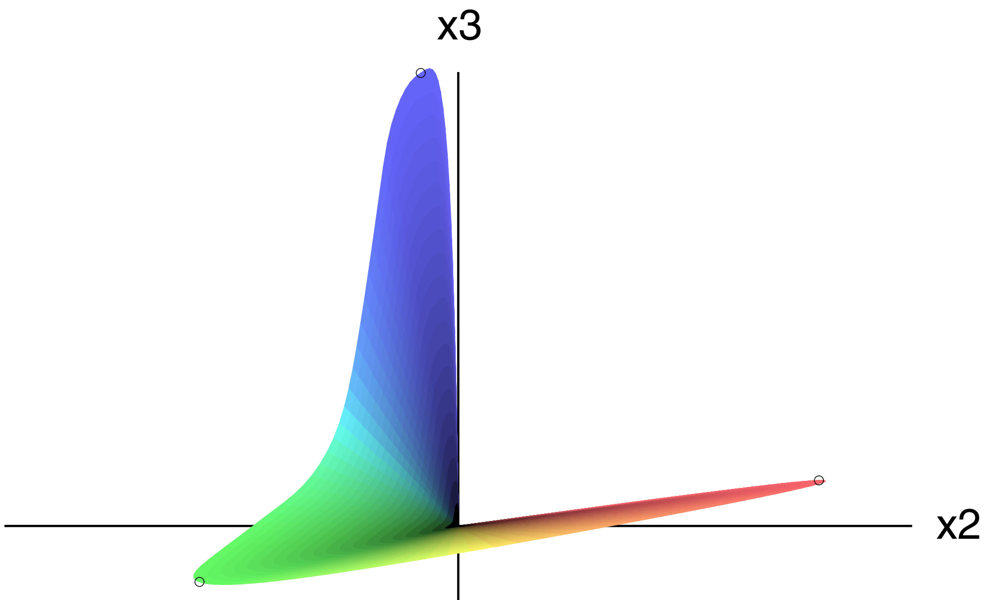
In the other diagrams the green curve is the second in the order R-G-B. It is the scaled CIE V(λ) curve. Here it is the first. The coordinate system is still right-handed, but the direct comparison is a little disturbed by the swapped R-B-axes (for simplicity all curves were called R,G,B, referring to the appearance).

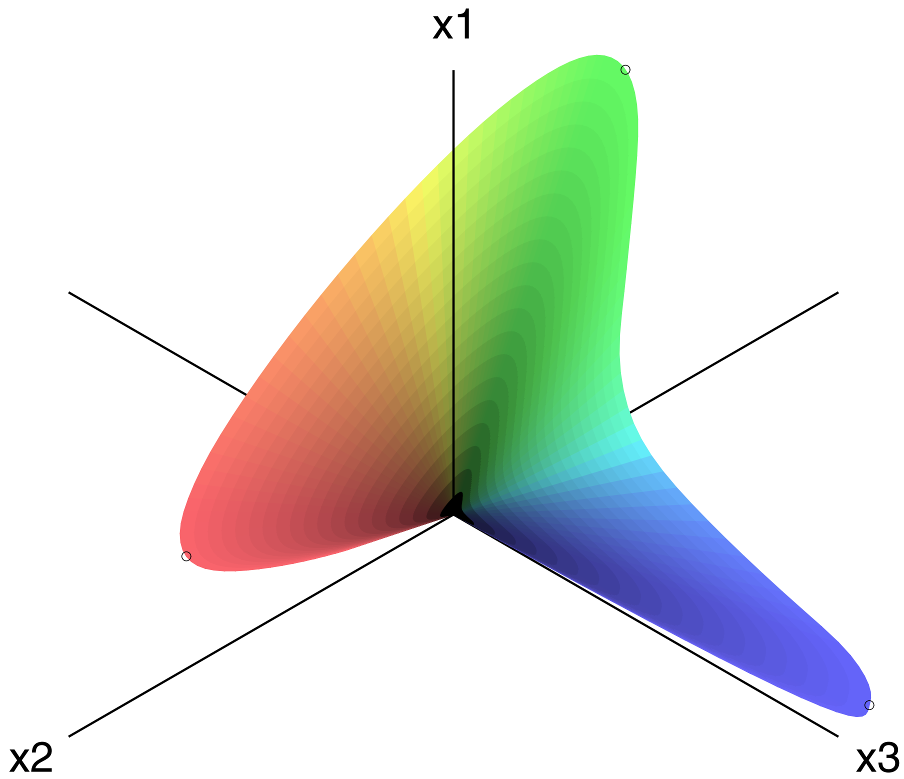
$$\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix} = \begin{bmatrix}0.0 & 0.11381 & 0.0\\0.18756 & -0.13328 & -0.03971\\0.03062 & -0.03121 & 0.07931\end{bmatrix}\begin{bmatrix}\bar{x}\\\bar{y}\\\bar{z}\end{bmatrix}$$









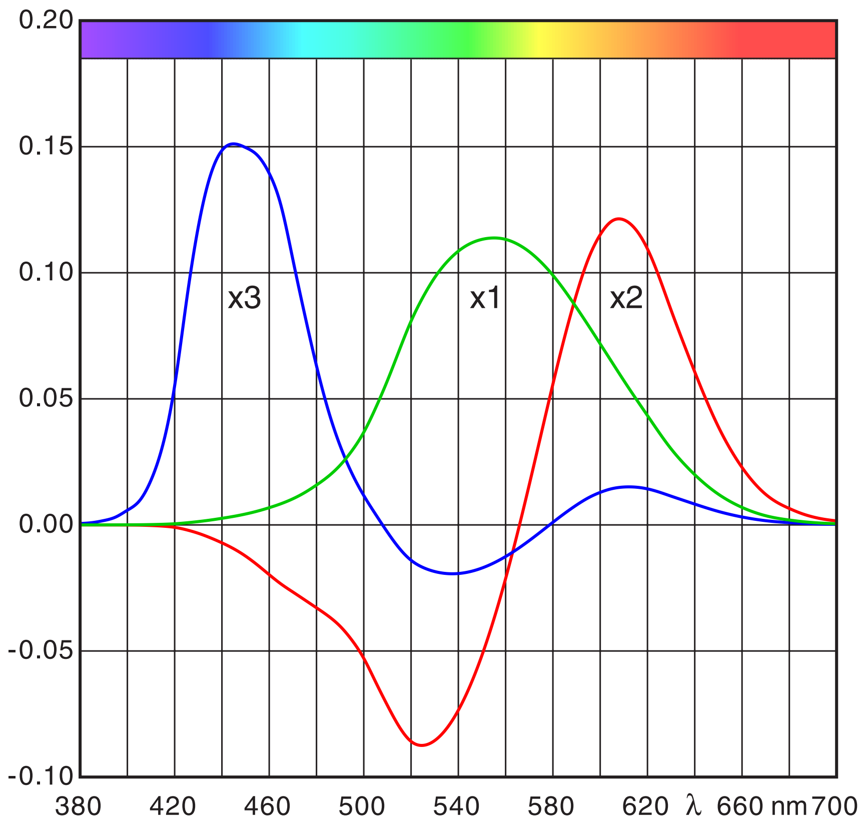


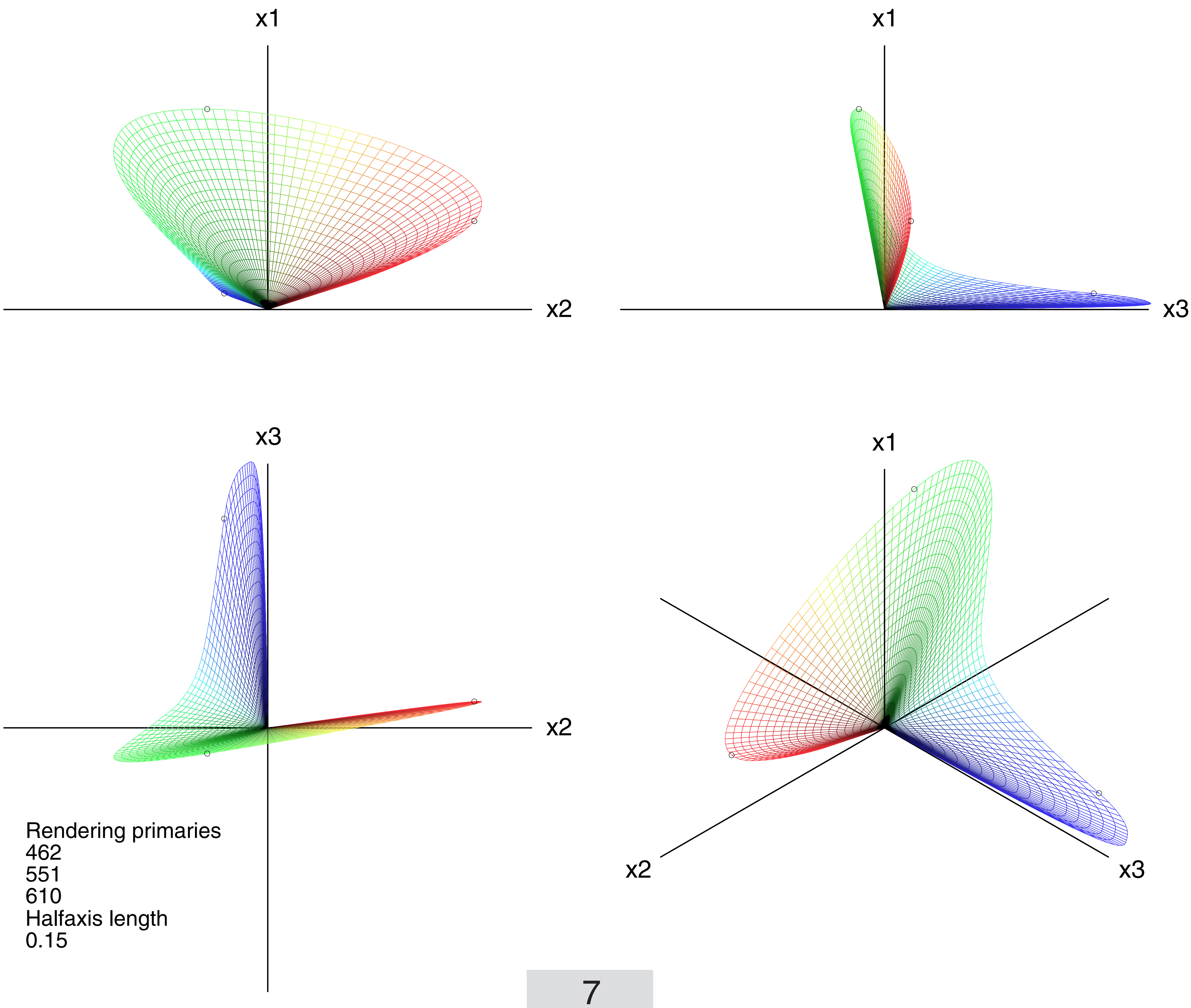
Rendering primaries
445
525
608
Halfaxis length
0.15

5.2 Worthey's Orthonormal Color-Matching Functions

Similar to previous page, but rendered by sRGB primaries.

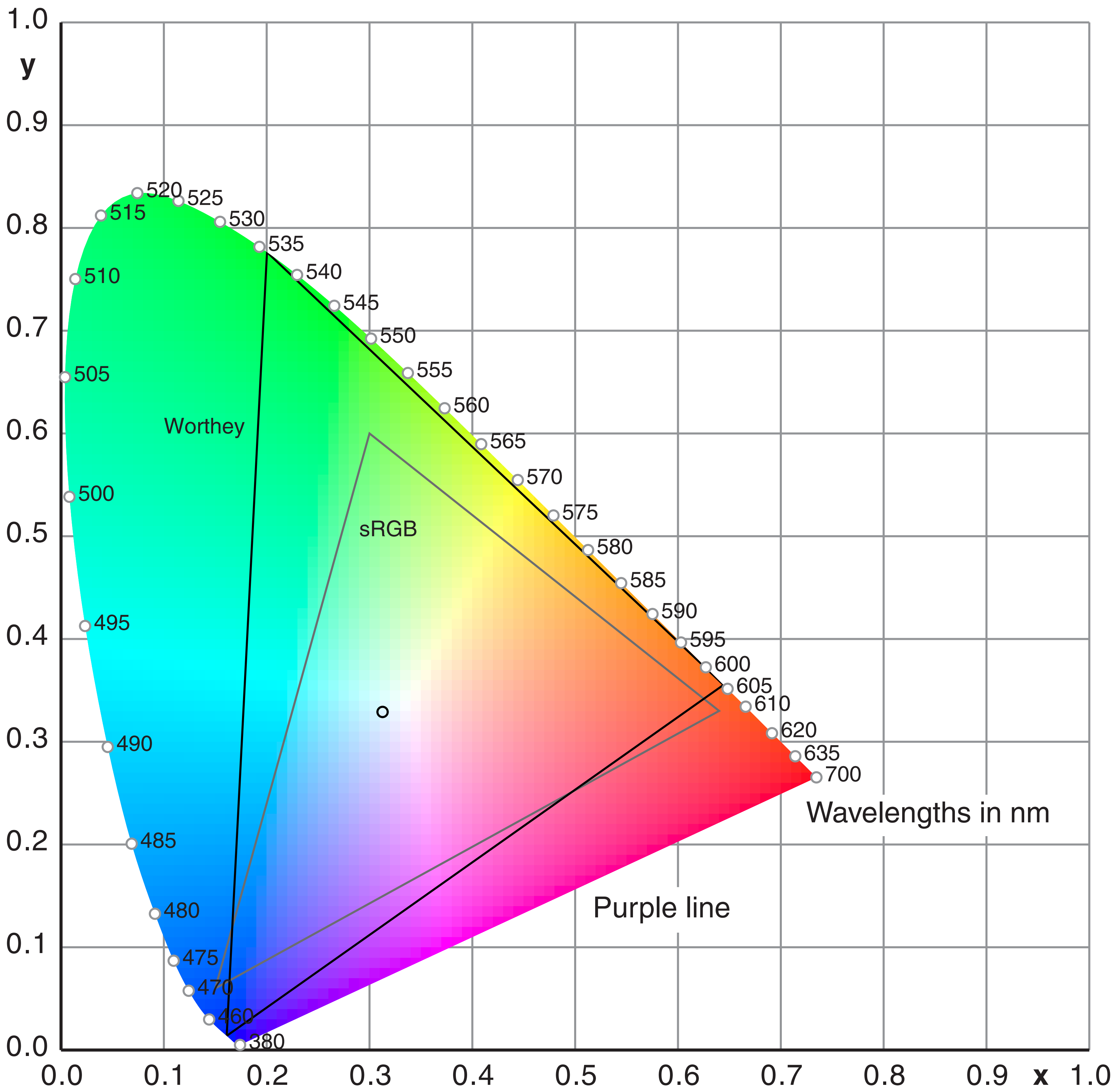
$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0.0 & 0.11381 & 0.0 \\ 0.18756 & -0.13328 & -0.03971 \\ 0.03062 & -0.03121 & 0.07931 \end{bmatrix} \begin{bmatrix} \bar{x} \\ \bar{y} \\ \bar{z} \end{bmatrix}$$





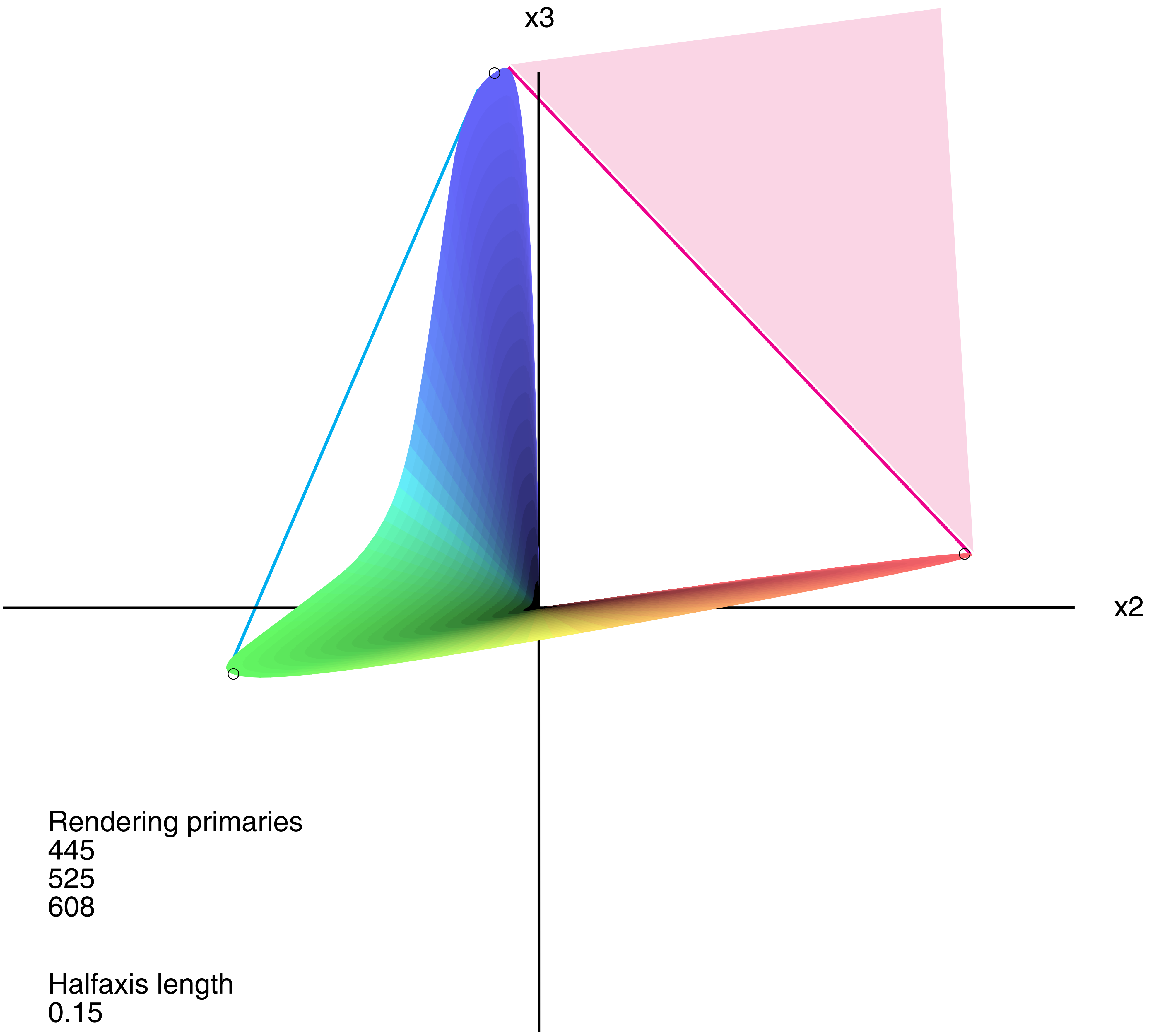
6. Optimal Primaries

According to J.A.Worthey's publication one can find 'optimal' primaries. This approach is based on 'amplitude not left out'. Which primaries should be used if the power is limited for each light source, e.g. for unit irradiance power?
The resulting wavelengths are found at the 'wing tips' of the surface for orthonormal color-matching functions.
Here they are shown by the corners of the triangle in the chromaticity diagram: 445, 536, 604 nm.
The wavelengths should be at least near to these values.
For any real system (besides expensive tests in a laboratory) pure spectral colors cannot be used. The corners have to be shifted on a radius towards the white point (which is here indicated by the circle for D65).
The location of the optimal primaries differs rather much from real monitors and working spaces like sRGB and AdobeRGB(98).
This raises a question: can unit power irradiance for the optimal primaries be generated by physical light sources with the same power *input* ? In other words: is it technically possible to create an illumination system with such a strong blue primary ?



7. Cyan and Purple

The projection of the surface onto the x_2-x_3-plane delivers a kind of chromaticity diagram.
We can see that the purple line of the CIE chromaticity diagram, which connects approximately 380nm and 700nm on the spectral locus by a straight line, is practically shrinked to a point at the origin. Both monochromatic light sources have the same unit power like all the others, but they are alomost not perveivable.
J.A.Worthey defines the ‘line of practical purples’ and the ’line of practical cyans’ as shown below.
This would be so if we *interpolate* a magenta color **m** linearly by **m**=**r**(1-t) + **b**t, using primaries **r**,**b** and a parameter t= 0 to 1.
But before it was said that each primary is available by unit power. Therefore the *mixing* could be done by **m**=**r**s + **b**t with two parameter s=0 to 1 and t= 0 to 1. This would add an additional area as shown for magenta.



Rendering primaries
445
525
608

Halfaxis length
0.15

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June 12 / 2004 — October 14 / 2013
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