Please use 72dpi and zoom 100%

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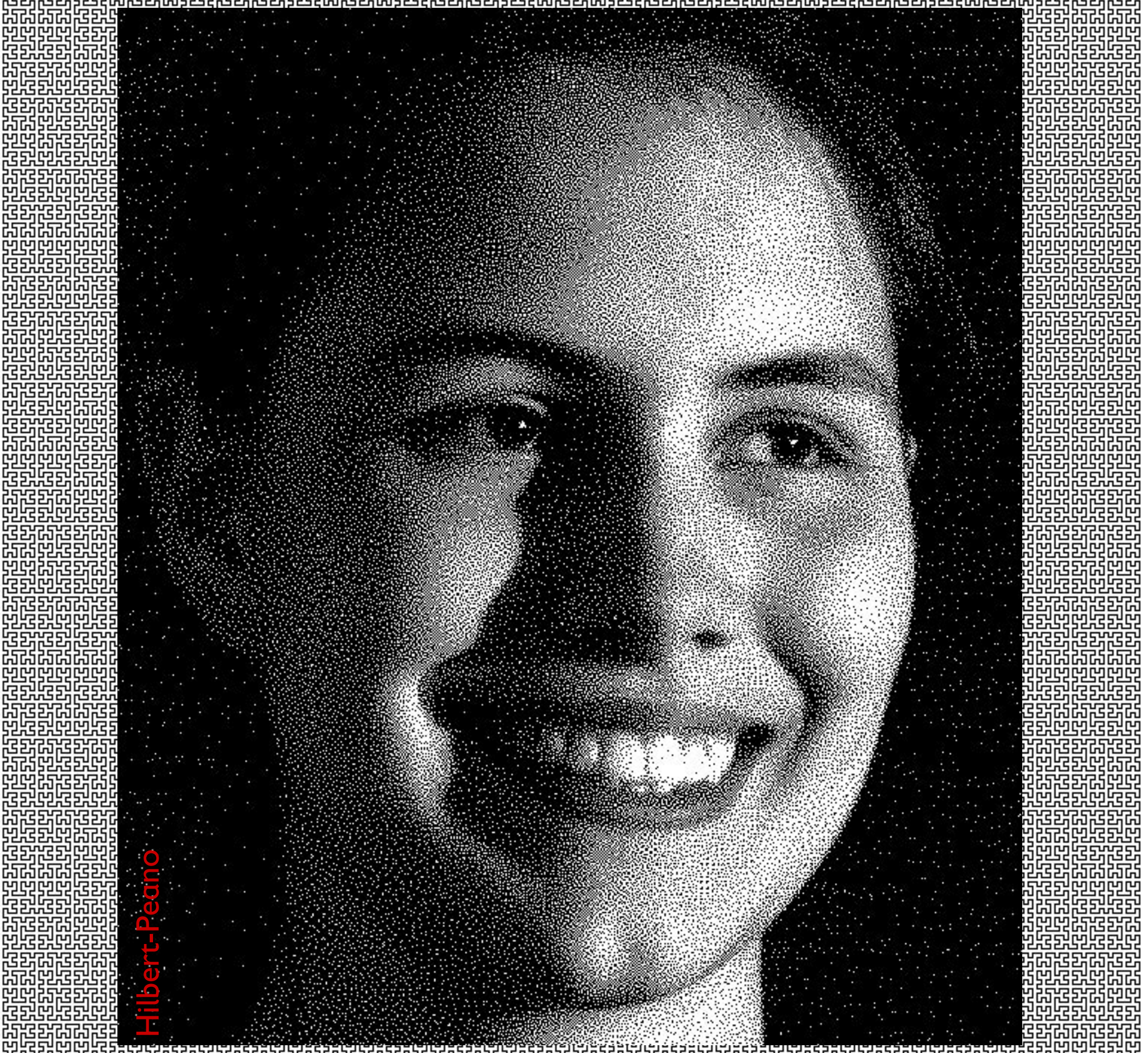
1. 1 Dithering by Hilbert-Peano

Based on an idea of Peano, David Hilbert had published 1891 the mathematical description of a function, which maps all points of a plane to the points of a line.
This curve has some features which are very helpful for dithering images:
- The curve walks always in her own vicinity
- The curve hits all points (dithering dots) in the plane exactly once
- The curve seems to walk on a random path

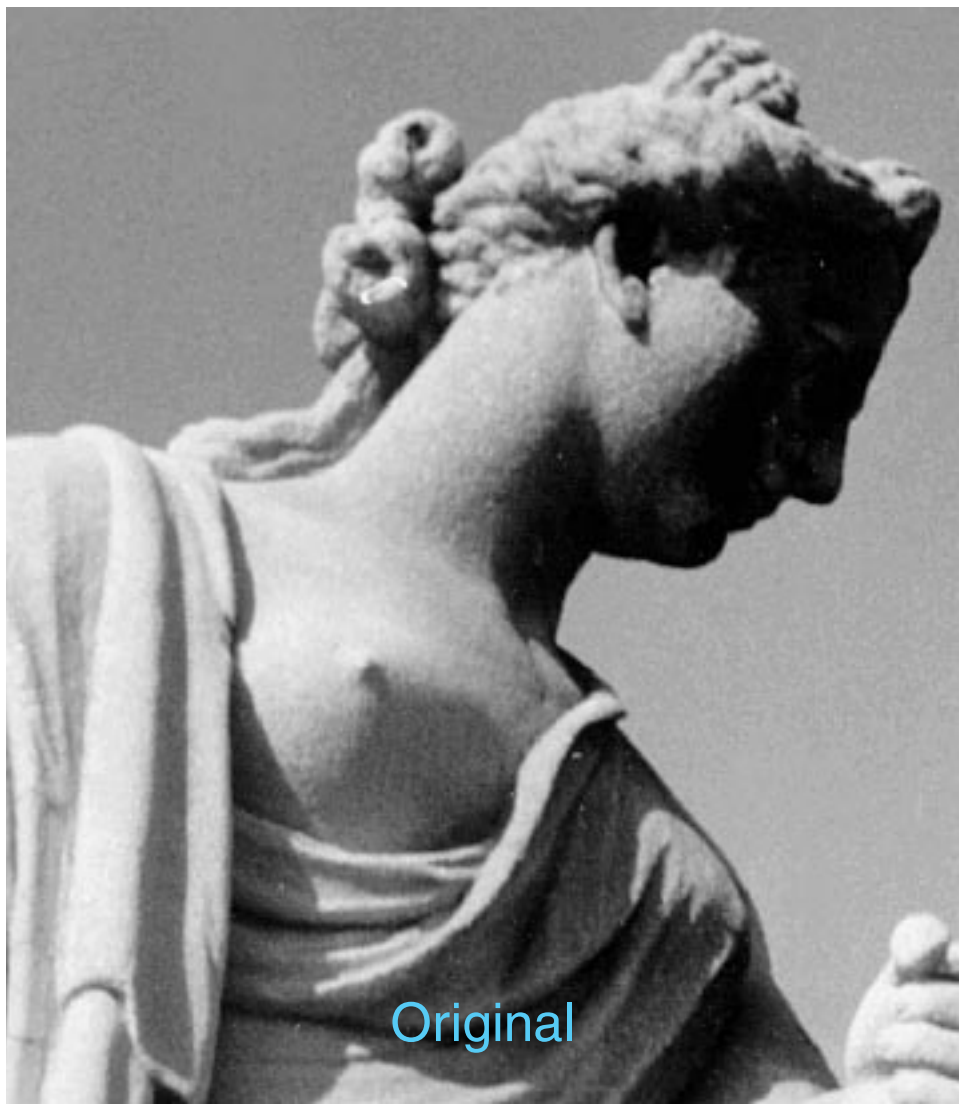
Along the path all color values, e.g. cyan, are accumulated. If the sum is greater than 255, a cyan point is printed and in the accumulator 255 is subtracted. This is a bilevel printing: cyan or nothing. Multilevel printing with different dotsizes is possible as well.
The curve can be programmed by powers of 2. Here we have the width 256.

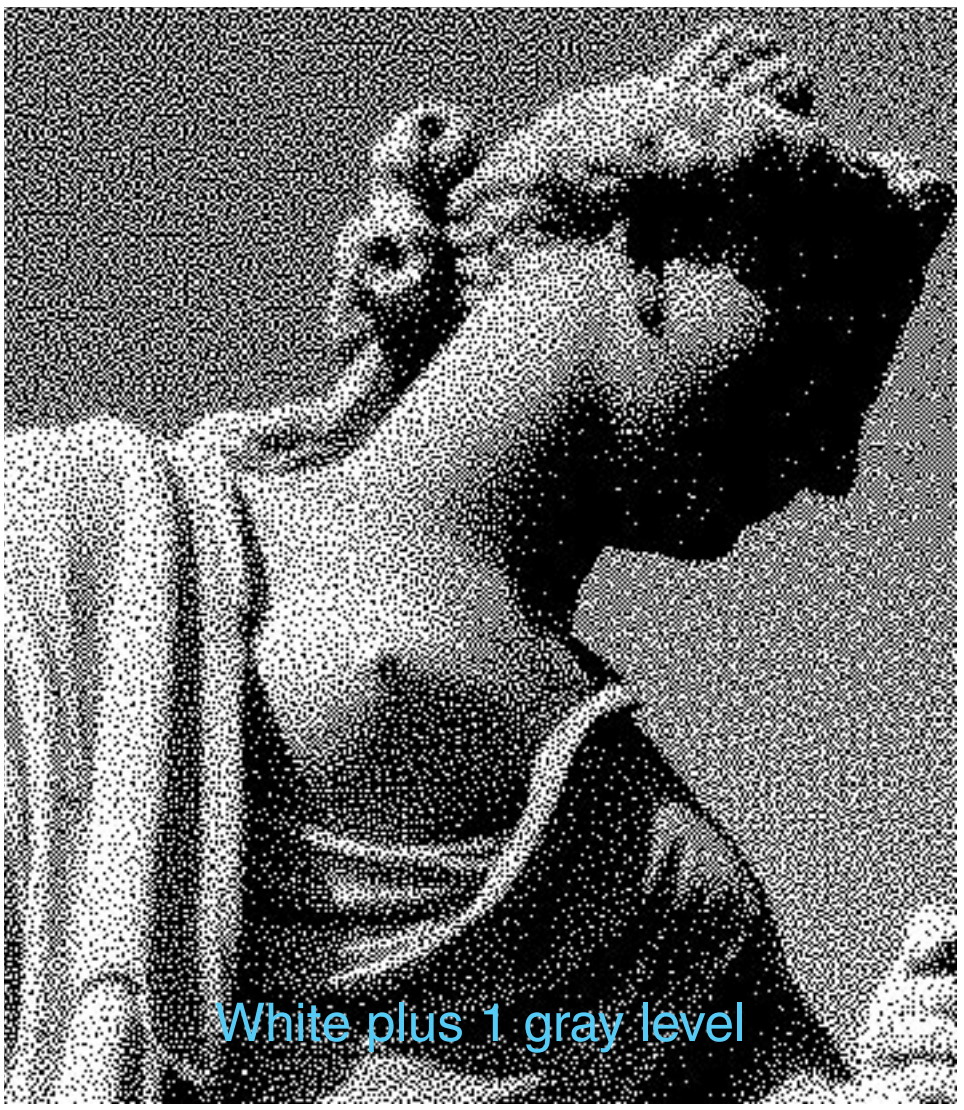


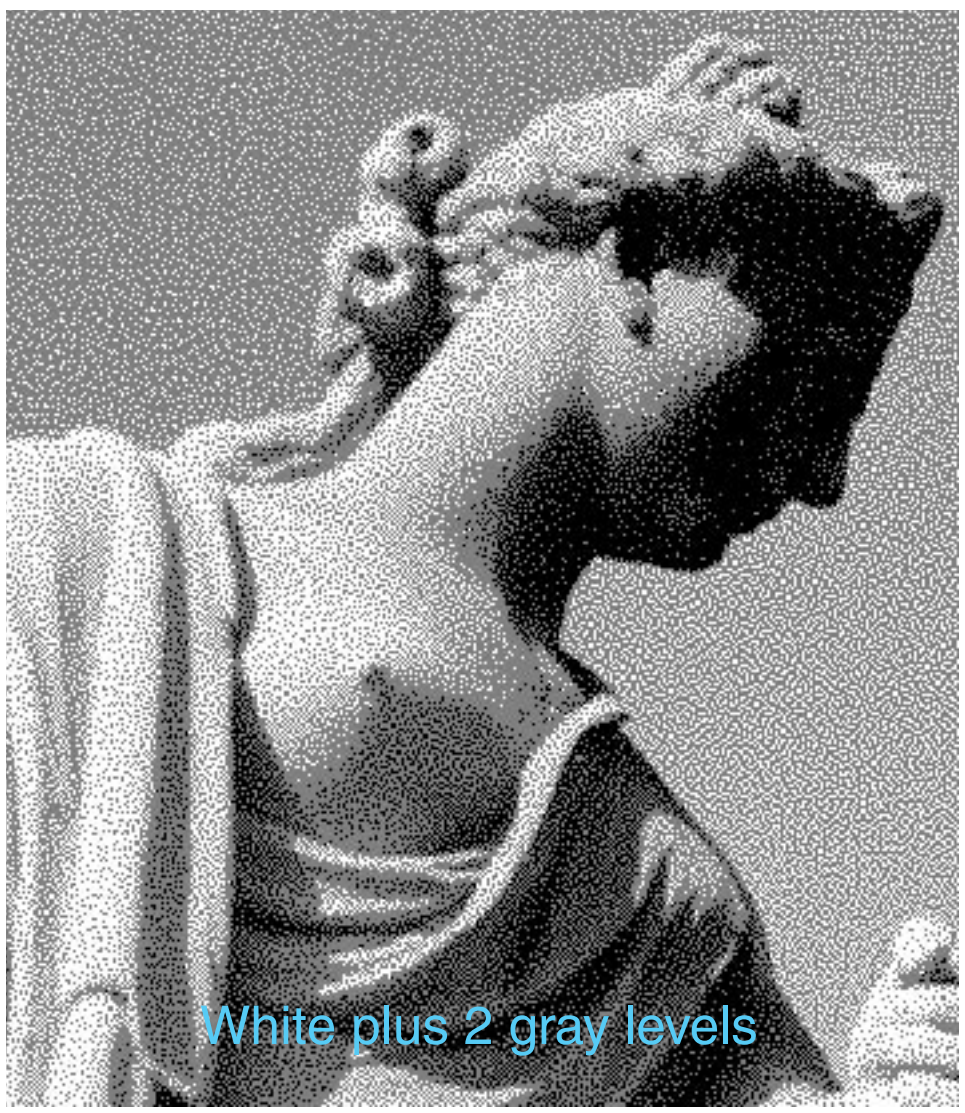
1.2 Dithering by Hilbert-Peano

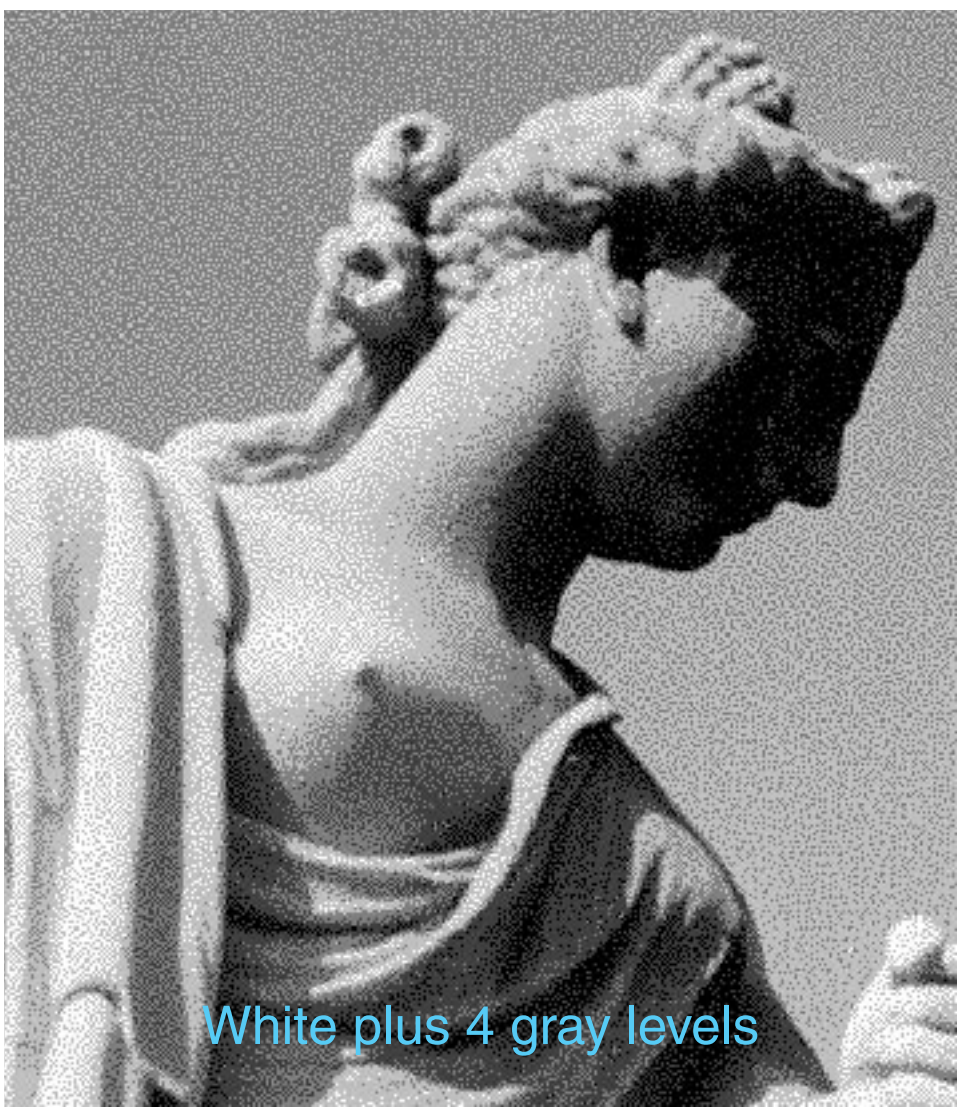
This page shows an original Grayscale Image and Hilbert-Peano Dithering with one, two and four gray levels.
This is a screen simulation. Instead of small printer dots we have large pixels.
Image Processing by ZEBRA.

For correct pixel view download and use Acrobat Reader Zoom 100% or 200%





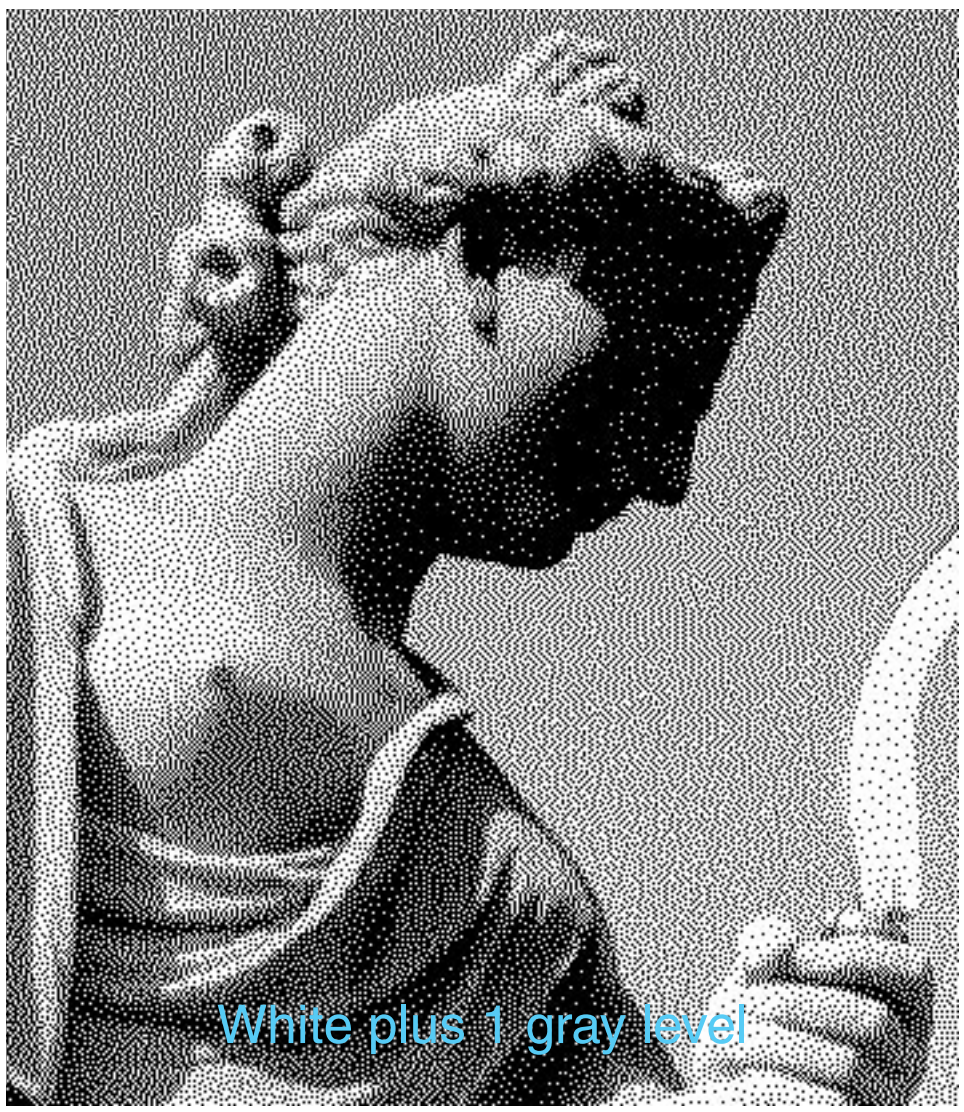


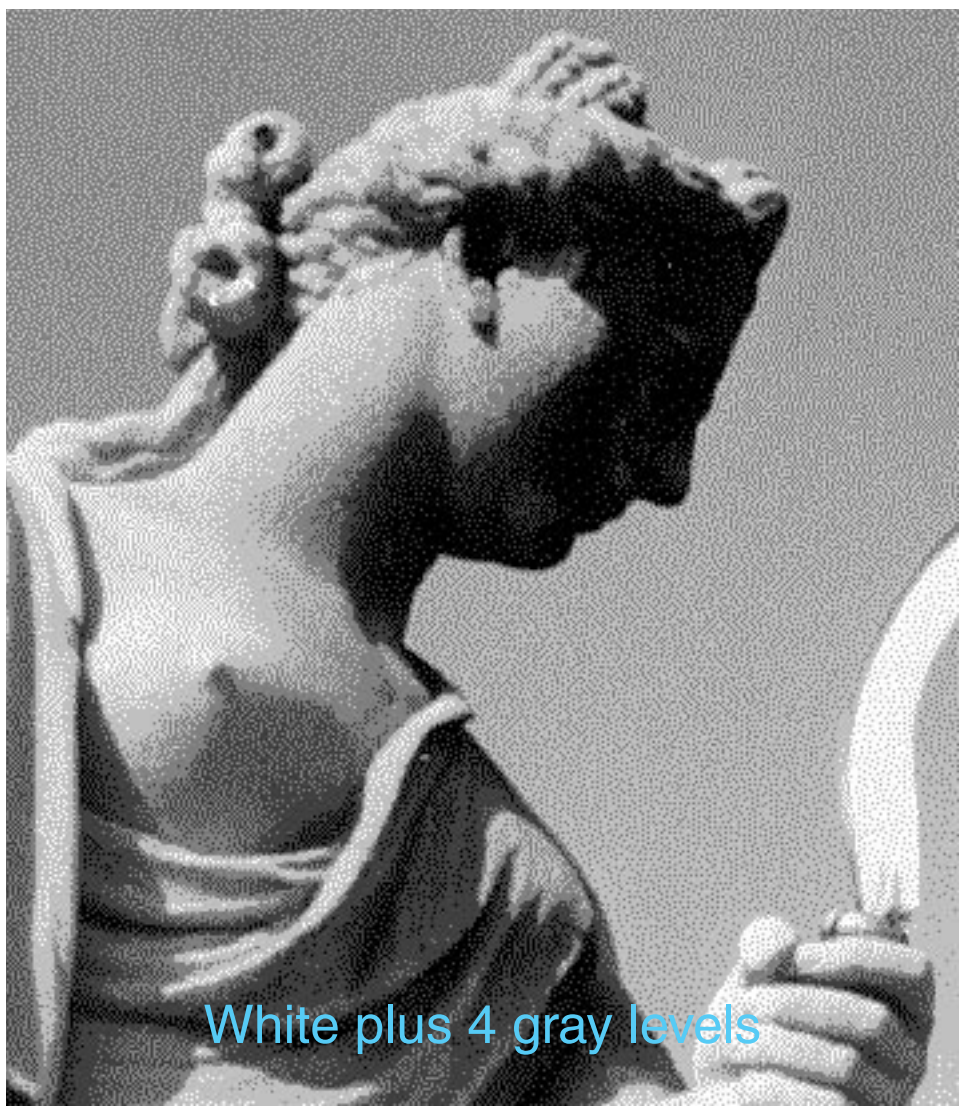


2.1 Dithering by Floyd-Steinberg

This page shows an original Grayscale Image and Floyd-Steinberg Dithering with one, two and four gray levels.
This is a screen simulation. Instead of small printer dots we have large pixels.
Image Processing by ZEBRA.

For correct pixel view download and use Acrobat Reader Zoom 100% or 200%





2.2 Dithering by Floyd-Steinberg

The Floyd-Steinberg algorithm was programmed as recommended in [3] with alternating directions for even and odd rows. This is the error weighting scheme for left to right dithering in even rows:

y=0	x=0										xmx
y	x	7/16			x	7/16					x
	5/16	1/16		3/16	5/16	1/16				3/16	5/16
ymx											

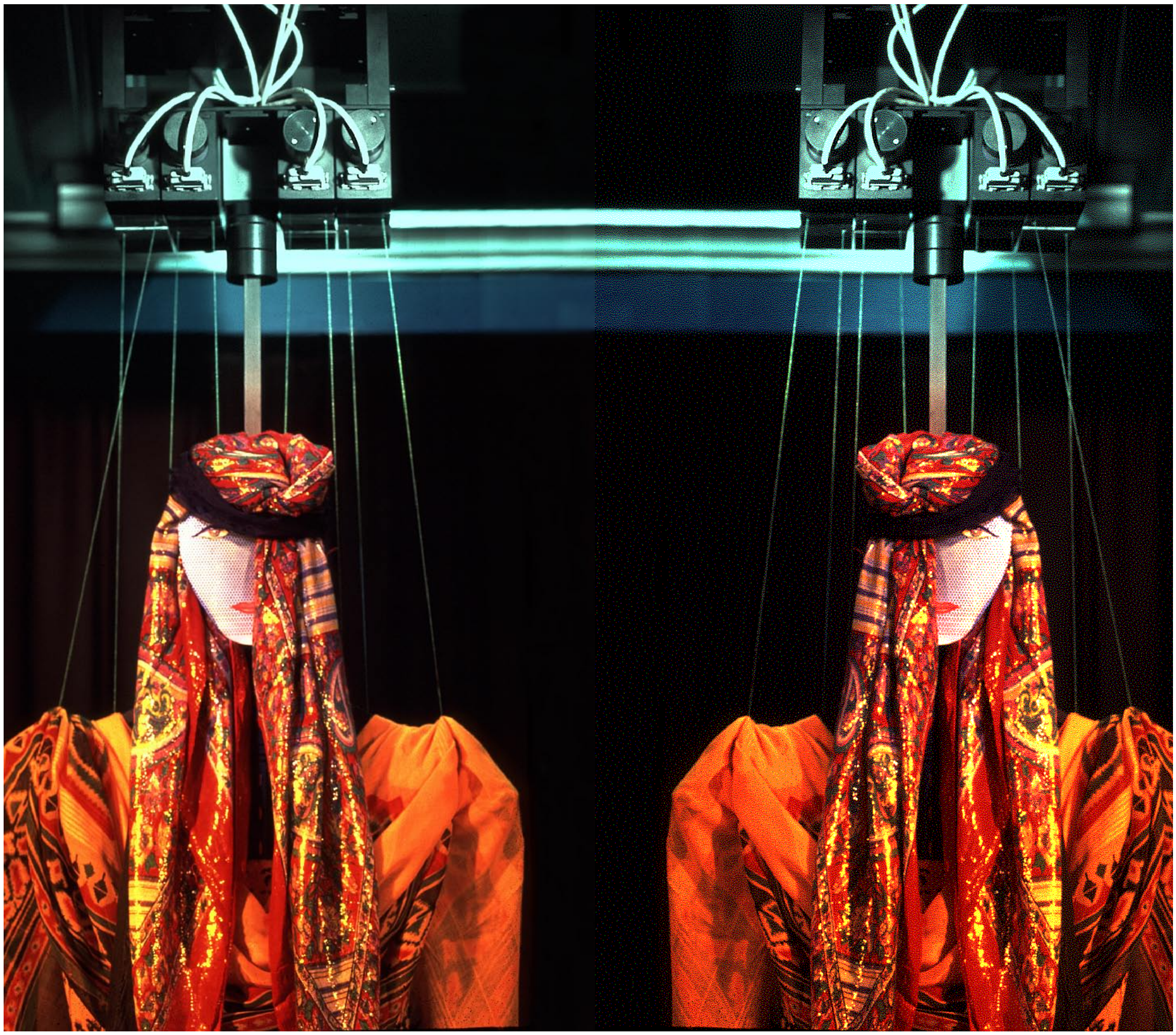
Bilevel Dithering: here each destination color C=R,G,B is either 0 or 255. Hilbert-Peano (page 3) delivers generally less sharp results, compared to Floyd-Steinberg (page 1, cover). Artifacts may result in synthetical images in uniform or shaded areas, but nearly never in photos. The Hilbert-Peano algorithm is based on [1] and [2].

Floyd-Steinberg creates also artifacts in synthetical images. Sometimes it is helpful to dither first trilevel and then bilevel.

For monitor simulations of bilevel dithering it is generally necessary to cancel the working space gamma to some extent. Recalculate all source colors C=R,G,B by $C=C^G$. Gamma is not necessarily 2.2. Sometimes it is better to use 1.6.

If images with bilevel dithering or halftoning are converted to smooth images by blurring filters then it is necessary to apply an inverse gamma correction by $C=C^{1/2.2}$ for 2.2 working spaces. This has nothing to do with the gamma correction in advance to the dithering or halftoning.

2.3 Dithering by Floyd-Steinberg



For correct pixel view download and use Acrobat Reader zoom 200%

The Floyd-Steinberg algorithm can be used for color quantization.

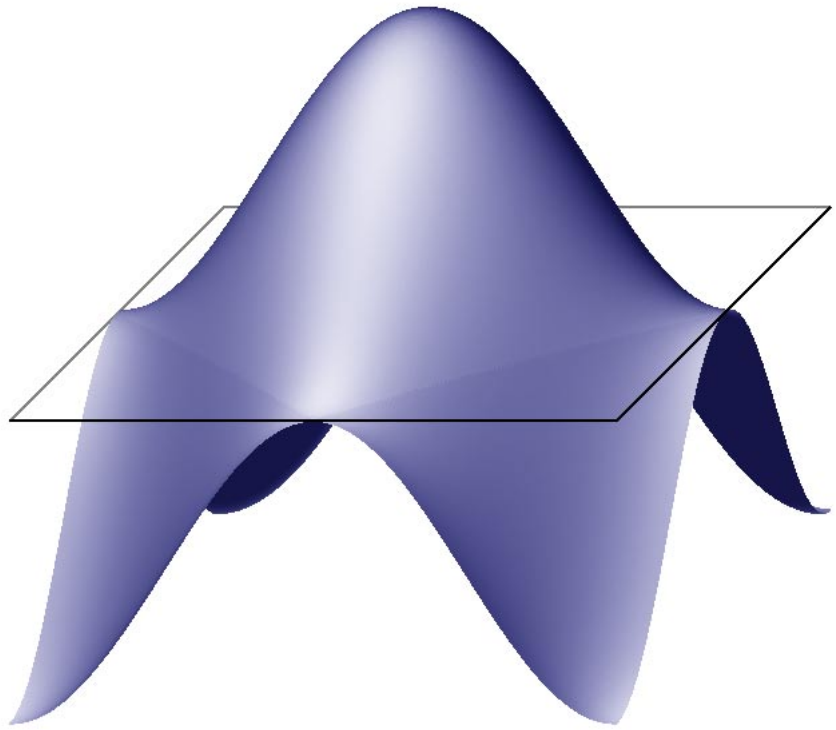
Left: original image.

Right: three levels for each channel R,G,B .

3.1 Raster Halftoning

This is a tutorial demonstration of raster halftoning. It is used for offset or laser printing, either for gray images or for each color channel separately.
Each raster cell contains one spot. A spot consists of printable dots with device resolution. We don´t call these dots pixels. Raster cells are arranged in Lpi distance, dots in dpi distance. A square raster cell with n dots in one direction can reproduce $m=n^2$ levels plus white.
The image is virtually placed on the paper. A frame with raster cell width is placed on the paper. For each device dot the underlying source image pixel value is taken and compared with a spot function.
In a previous version of this doc it was assumed that the gray values in the frames are averaged. This assumption was wrong.

We are starting with a simplified explanation: The gray in a uniform area defines the height for the spot function. Everything inside the height contour has to be filled by black.
Light gray would deliver a nearly round spot. 50% gray a square spot and dark gray four white quarter circles at the corners which appear as round white spots on the printed paper. Black and white dots are complementary to each other with respect to medium gray 50%.



Actually it works differently. Quoted by [4], where we have replaced 'pixel' by 'dot':
'The values the spot function returns are not significant. All that matters is the *relative* spot function values for different dots. A cell´s gray value varies from black to white, the first dot whitened is the one whose spot function has the lowest value, the next pixel is the one with the next higher spot function value, and so on. If two dots have the same spot function value, **setscreen** chooses their relative order arbitrarily.'

One may think that each cell has *one* gray. As already mentioned this is wrong. Nothing is averaged - the underlying image pixels are directly used, which preserves sharp image elements but may cause fragments of spots.
The spot function can have any shape and height, it is by no means necessary to normalize the function, let us say for height one. Of course the spot function is defined for one raster cell.

The mechanism becomes better understandable by the introduction of threshold arrays.

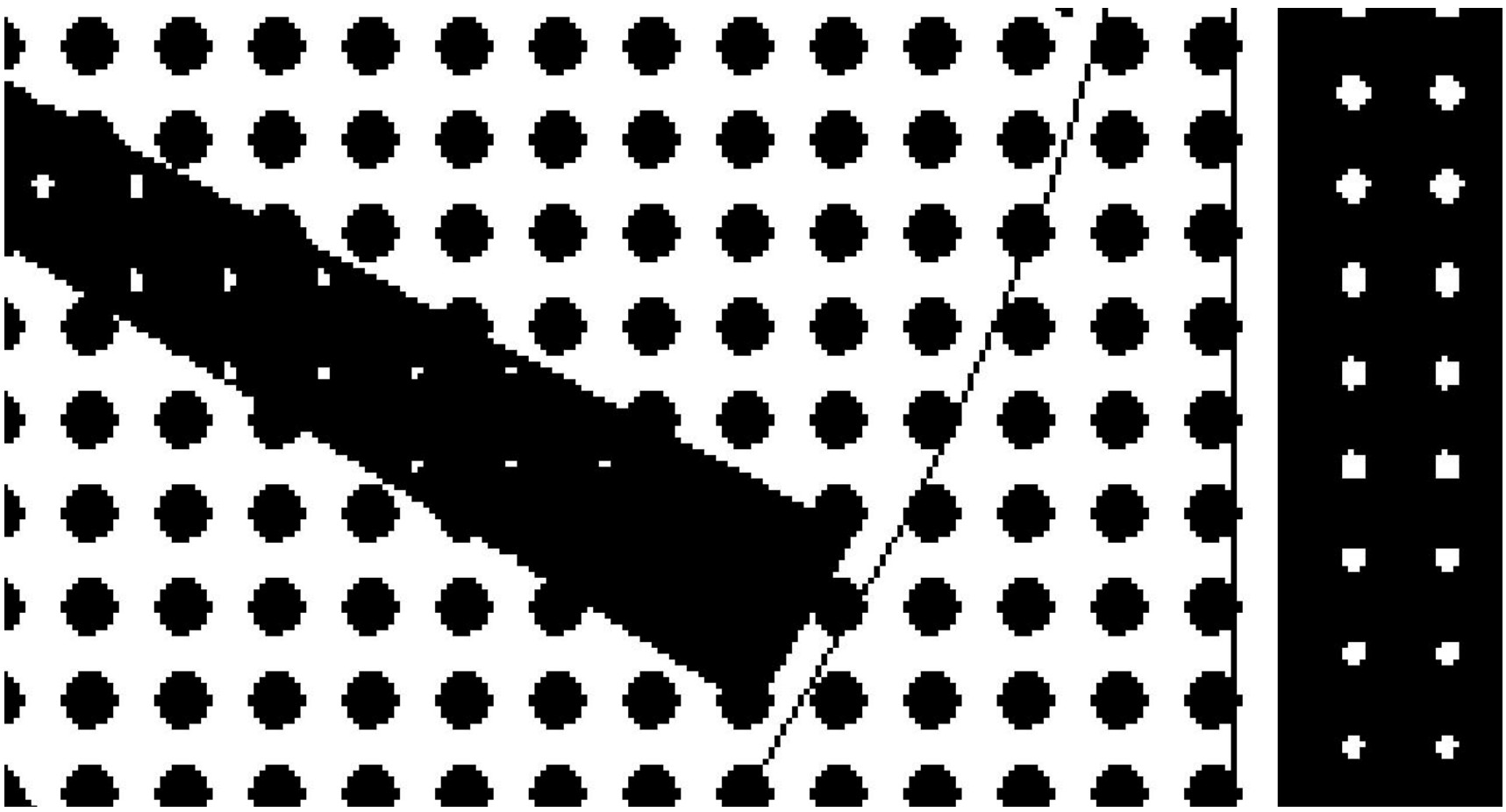
3.2 Raster Halftoning

The image (right) shows a threshold array for n=6. A dot is drawn black if the generating pixel delivers a gray value below the threshold. The threshold arrays are here created by spot functions. A float increment is calculated by dg=1/m with $m=n^2$. The first decision level is 1-dg/2. The other levels are generated by decrementing. The lowest decision level is dg/2. The levels are multiplied by 255 and rounded.

```
C:\ZESNP\ZESNP001.BMP
36 Levels   Increment 7.083
 25  81 166 152  67  18
 53 110 223 216 103  46
138 195 251 244 181 131
124 188 237 230 174 117
 39  96 209 202  89  32
  4  74 159 145  60  11
```

The threshold array is filled by zeros.The raster cell is scanned on a spiral contour which starts in the center for *odd* n or with a half step offset left and up for *even* n. This is the strategy: find all values of the spot function which are not yet used and which are not larger than the previous value. Then find the maximum of these. This delivers a new entry for the array. Decrement the level. Execute the scan m times.

For n=16 the lowest threshold would be zero. This has to be replaced by 1, because dots are drawn black if the gray value is *below* the level.



The image shows the rasterization for n=16 in a synthetical graphic

Screens with non-zero angle are created by a brute force method: rotate the image, rasterize the whole area and rotate back using nearest neighbour 'interpolation'. This works reasonably for 45° and for larger n. Better methods can be found in [5].

In the example we have n=16 dots per cell width. The fuzzyness of the spots could be reduced by blurring the source image a little. This holds true mainly for photos.

3.3 Raster Halftoning Example

C:\ZESNP\ZESNP001.BMP
36 Levels Increment 7.083
25 81 166 152 67 18
53 110 223 216 103 46
138 195 251 244 181 131
124 188 237 230 174 117
39 96 209 202 89 32
4 74 159 145 60 11

Zebra Sub Menue
J:\\GIESE004.BMP BMP-32
Raster Halftone
↑↓ 6.00 6.00 Pixel
D Dot active
S Square
H Horiz-Line
V Verti-Line
E Experiment
Q/A Angle = 0
N/M Gamma = 2.20
Δ∇ Threshold M.= Show
< Undo Raster
ESC Menue
X Shift Menue Area

n=6, 36 levels, angle 0°
Use zoom 200%

3.4 Raster Halftoning Example

n=16, 256 levels, angle 45°
Use zoom 200%

4.1 Code Floyd-Steinberg

```
Procedure IFloyd(ncol: Integer);
{    Copyright Gernot Hoffmann; November 20, 2001
     ncol: Color levels: white + 1,2,4,8,16,32,64,128
     Floyd-Steinberg
     Source FImage; Destination FImage; works inplace
     Uses global    FImage=FMem[y]^[x]; packed prgb=pb/rb/gb/bb
                    gmx : for x=0..gmx  Pixels, gmx<=1279
                    gmy : for y=0..gmy  Pixels
    This is an edited program text.     Errors cannot be excluded }

Var  x,y,i,j,xa,ya,xe,ye,flap   :   Integer;
     pb                         :   Byte;
     ri,gi,bi,er,k1,k2,k3,c1,c2:   Word;
     q,z,gam                    :   Single;
     r1,r2,g1,g2,b1,b2   : Array[-1..1280] Of Word;
     tabr,tabg,tabb      : Array[ 0.. 255] Of Byte;
Begin
xa:=0; ya:=0; xe:=gmx; ye:=gmy;
gam:=1.0;
Case ncol Of
 1: gam:=1.60;
 2: gam:=1.15;
End; { case }
For i:=0 to 255 Do
Begin q:=i/255;
XPowerA(q,gam,z     ,flap);    Tabr[i]:=Round(255*z);
XPowerA(q,gam+0.05,z,flap);    Tabg[i]:=Round(255*z);
XPowerA(q,gam,z     ,flap);    Tabb[i]:=Round(255*z);
End;
Case ncol Of
1: Begin c1:=256;   c2:=8; End; {  1 color  }
2: Begin c1:=128;   c2:=7; End; {  2 colors }
3: Begin c1:= 64;   c2:=6; End; {  4 colors }
4: Begin c1:= 32;   c2:=5; End; {  8 colors }
5: Begin c1:= 16;   c2:=4; End; { 16 colors }
6: Begin c1:=  8;   c2:=3; End; { 32 colors }
7: Begin c1:=  4;   c2:=2; End; { 64 colors }
8: Begin c1:=  2;   c2:=1; End; {128 colors }
End;
For x:=xa to xe Do
Begin r1[x]:=0; r2[x]:=0; g1[x]:=0; g2[x]:=0; b1[x]:=0; b2[x]:=0;
End;
y:=ya;
Repeat
  For x:=xa to xe Do
 Begin
   GetFixel(x,y,pb,ri,gi,bi);

   er:=r1[x]+Tabr[ri];              er:=g1[x]+Tabg[gi];           er:=b1[x]+Tabb[bi];
   ri:=(er Div c1)*c1;              gi:=(er Div c1)*c1;           bi:=(er Div c1)*c1;
   If ri>255 Then ri:=255;          If gi>255 Then gi:=255;       If bi>255 Then bi:=255;
   er:=er-ri;                       er:=er-gi;                    er:=er-bi;
   k1:=(7*er)Div 16;                k1:=(7*er)Div 16;             k1:=(7*er)Div 16;
   k2:=(3*er)Div 16;                k2:=(3*er)Div 16;             k2:=(3*er)Div 16;
   k3:=(5*er)Div 16;                k3:=(5*er)Div 16;             k3:=(5*er)Div 16;
   r1[x+1]:=r1[x+1]+k1;             g1[x+1]:=g1[x+1]+k1;          b1[x+1]:=b1[x+1]+k1;
   r2[x-1]:=r2[x-1]+k2;             g2[x-1]:=g2[x-1]+k2;          b2[x-1]:=b2[x-1]+k2;
   r2[x  ]:=r2[x  ]+k3;             g2[x  ]:=g2[x  ]+k3;          b2[x  ]:=b2[x  ]+k3;
   r2[x+1]:=r2[x+1]+er-k1-k2-k3;    g2[x+1]:=g2[x+1]+er-k1-k2-k3; b2[x+1]:=b2[x+1]+er-k1-k2-k3;

   SetFixel(x,y,pb,ri,gi,bi);
 End;
For i:=xa to xe Do
 Begin
   r1[i]:=r2[i]; g1[i]:=g2[i]; b1[i]:=b2[i]; r2[i]:=0; g2[i]:=0; b2[i]:=0;
 End;
```

4.2 Code Floyd-Steinberg

```
Inc(y); If y>ye Then Exit;

 For x:=xe DownTo xa Do
  Begin
    GetFixel(x,y,pb,ri,gi,bi);

    er:=r1[x]+Tabr[ri];           er:=g1[x]+Tabg[gi];           er:=b1[x]+Tabb[bi];
    ri:=(er Div c1)*c1;           gi:=(er Div c1)*c1;           bi:=(er Div c1)*c1;
    If ri>255 Then ri:=255;       If gi>255 Then gi:=255;       If bi>255 Then bi:=255;
    er:=er-ri;                    er:=er-gi;                    er:=er-bi;
    k1:=(7*er)Div 16;             k1:=(7*er)Div 16;             k1:=(7*er)Div 16;
    k2:=(3*er)Div 16;             k2:=(3*er)Div 16;             k2:=(3*er)Div 16;
    k3:=(5*er)Div 16;             k3:=(5*er)Div 16;             k3:=(5*er)Div 16;
    r1[x-1]:=r1[x-1]+k1;          g1[x-1]:=g1[x-1]+k1;          b1[x-1]:=b1[x-1]+k1;
    r2[x+1]:=r2[x+1]+k2;          g2[x+1]:=g2[x+1]+k2;          b2[x+1]:=b2[x+1]+k2;
    r2[x  ]:=r2[x  ]+k3;          g2[x  ]:=g2[x  ]+k3;          b2[x  ]:=b2[x  ]+k3;
    r2[x-1]:=r2[x-1]+er-k1-k2-k3; g2[x-1]:=g2[x-1]+er-k1-k2-k3; b2[x-1]:=b2[x-1]+er-k1-k2-k3;

     SetFixel(x,y,pb,ri,gi,bi);
   End;
  For i:=xa to xe Do
   Begin
   r1[i]:=r2[i]; g1[i]:=g2[i]; b1[i]:=b2[i]; r2[i]:=0; g2[i]:=0; b2[i]:=0;
   End;
 Inc(y);
 Until y>ye;
End; { IFloyd }
```

5.1 Code Raster Halftoning

```
These Procedures are not intended to be copied because the underlying graphics library is
not available in Borland Pascal.

Procedure ISpotT(rwid,dot,alf,tmat: Integer; gam: Single);
{ G.Hoffmann
  January 07, 2004
  Convert Color to Black on White
  Rasterize by Threshold Matrix
  FMem   Works inplace in global framebuffer FMem
  rwid   Width of halftone cell in pixels, 2..16
  dot=1  Round
  dot=2  Square ...  }
Var  x,y,i,j,k,p,m0,m1,mq,si,sj    : Integer;
     xa1,ya1,xe1,ye1,xm1,ym1,xi,yj : Integer;
     flag,n,kmax                  : Integer;
     xa4,ya4,xe4,ye4               : Integer;
     pp,mm,so,sn,sm,gry,dgr,x1,y1  : Single;
     prgb,pbbb,pwww                : LongInt;
     pb,rb,gb,bb                   : Byte;
     Pout                          : Boolean;
     txt1,txt2                     : String;
     Tab    : Array [0..255]      Of Byte;
     Thr    : Array [0..15,0..15] Of Byte;
     Procedure Dots;
     Begin
     If Not Pout And (Thr[i,j]=0) Then
     Begin
     { Function has maximum in center of raster cell
       or at the edges of the raster cell           }
       Case dot Of
              1: sn:=  coc(pp*i-pi)+0.90*coc(pp*j-pi); { Dot, coc=Cosine }
              2: sn:=-(Abs(i-mm)+0.90*Abs(j-mm));      { Square }
              3: sn:= -Abs(j-mm)-0.15*Abs(i-mm);       { x-Line }
              4: sn:= -Abs(i-mm)-0.15*Abs(j-mm);       { y-Line }
              5: sn:=-(Sqr(i-mm)+0.90*Sqr(j-mm));      { Circle }
       End; { Case}
       If sn<=so Then
       Begin
       If sn>sm Then
        Begin
         si:=i; sj:=j; sm:=sn;
        End;
       End;
     End;
    End;
Begin
{ Gamma correction gam=1.4..2.2 for 2.3 working space }
For i:=0 to 255 Do
Begin XPowerA(i/255,gam,gry,flag); Tab[i]:=Round(255*gry);
End;
{ Define white and black in 4-byte coding }
pwww:=255 SHL 16 + 255 SHL 8 +255;  { white 0 255 255 255 }
pbbb:=  1;                          { black 1   0   0   0 }
If rwid< 2 Then rwid:= 2;
If rwid>16 Then rwid:=16;
m0:=rwid-1;
m1:=rwid;            { width of cell                   }
mq:=Sqr(m1);         { number of levels, without white }
kmax:=mq;
mm:=0.5*m0;          { center, zero based              }
dgr:=1/mq;           { Increment in range 0..1         }
pp:=pi*2/m0;
pwww:=255 SHL 16 + 255 SHL 8 +255;  { white }
pbbb:=  1 SHL 24;                   { black }
{ Build threshold array }
For j:=0 to m0 Do
For i:=0 to m0 Do Thr[i,j]:=0;
```

5.2 Code Raster Halftoning

```
so:=+1E6;
gry:=1-dgr/2;
For k:=1 to kmax Do
Begin
{ Spiral scan }
sm:=-1E6;
pout:=False;
If odd(m1) Then i:=m0 Div 2 Else i:=m1 Div 2 -1;
j:=i; Dots;
n:=1;
 Repeat
  For p:=1 to n Do Begin Inc(i); If i>m0 Then Pout:=True; Dots; End;
  For p:=1 to n Do Begin Inc(j); Dots; End;
  Inc(n);
  For p:=1 to n Do Begin Dec(i); If i<0  Then Pout:=True; Dots; End;
  For p:=1 to n Do Begin Dec(j); Dots; End;
  Inc(n);
 Until Pout Or (n>m1);
 Thr[si,sj]:=Round(255*gry);
 If Thr[si,sj]=0 Then Thr[si,sj]:=1;
 so:=sm;
 gry:=gry-dgr;
End;
{ Find actual image size in frame buffer }
FrameLim ('F',xa1,ya1,xe1,ye1,xm1,ym1);
{ Gray Copy
  1=1/3 weight; 2=NTSC; 3=PhS            }
FMemGrYIQ (xa1,ya1,xe1,ye1,xa1,ya1,2);
ImiToScr; { Copy frame buffer to screen  }
{ Rotate image in frame buffer. Procedure is not in document }
RolTranP (+1,alf,rwid,xa1,ya1,xe1,ye1,xa4,ya4,xe4,ye4);
{ Rasterize }
y:=ya4;
Repeat
 x:=xa4;
 Repeat
 For j:=0 to m0 Do
 Begin
 yj:=y+j;
 For i:=0 to m0 Do
  Begin
  xi:=x+i;
 { Use blue channel of gray image via gamma table }
   bb:=FMem[yj]^[xi] AND $000000FF;
   If Tab[bb]<Thr[i,j] Then FMem[yj]^[xi]:=pbbb
                       Else FMem[yj]^[xi]:=pwww;
  End;
 End;
 x:=x+m1;
 Until x>xe4;
y:=y+m1;
Until y>ye4;
If (xa1<>0) Or (xe1<>gmx) Or (ya1<>0) Or (ye1<>gmy) Then
ColToImi(frab,whit); { Fill total area by white }
{ Rotate frame buffer back }
RolTranP(-1,alf,rwid,xa1,ya1,xe1,ye1,xa4,ya4,xe4,ye4);
End;
```

6.1 References

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[5] Henry R.Kang
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[6] About Color Rendering
http://docs-hoffmann.de/colrend290800.pdf
http://docs-hoffmann.de/spot290800.pdf

Important note:

This is a pixel synchronized document. The synchronization is correct for

Acrobat 72dpi + Zoom 100% or 200%

or

Acrobat 96dpi + Zoom 75% or 150%

Gernot Hoffmann
January 01 / 2001 – April 21 / 2019
Website
Load browser and click here